Dept. of Math. Sci., WPI
MA 1034 Analysis 4
Instructor: Bogdan Doytchinov, Term D01

Homework Assignment 1

Due Tuesday, March 25, 2001

1. Find an equation of a sphere that has a center at the point $(5, 3, -6)$ and touches the yz-plane.

2. Find the equation of the sphere with a diameter determined by its endpoints, $(1, -2, 7)$ and $(9, 0, 1)$.

3. Find the center and the radius of a sphere with an equation

$$2x^2 + 2y^2 + 2z^2 + 4y - 2z = 1.$$

4. Two spheres with equations

$$x^2 + y^2 + z^2 - 4x - 2y - 6z + 12 = 0$$

and

$$x^2 + y^2 + z^2 - 2x + 2y - 10z + 24 = 0$$

respectively, intersect. Their intersection is a circle in $\mathbb{R}^3$. Find the center and the radius of this cirle.

5. Write the vector $\vec{u} = \langle 1, 0, -2\rangle$ as a linear combination of the three vectors $\langle 1, 1, 0\rangle$, $\langle -1, 1, -2\rangle$, and $\langle 0, 1, -2\rangle$.

6. Let $\|\vec{a} + \vec{b}\| = \|\vec{a} - \vec{b}\|$. Find the angle between $\vec{a}$ and $\vec{b}$.

7. The Paralleloram Law states that

$$\|\vec{a} + \vec{b}\|^2 + \|\vec{a} - \vec{b}\|^2 = 2\|\vec{a}\|^2 + 2\|\vec{b}\|^2.$$

Give a geometric interpretation of this law. Prove it.

8. If $\|\vec{a}\| = \|\vec{a} - \vec{b}\|$, show that $\vec{a} \cdot \vec{b} = (\vec{b} - \vec{a}) \cdot \vec{b}$.

9. If $\vec{c} = \|\vec{a}\|\,\vec{b} + \|\vec{b}\|\,\vec{a}$, where $\vec{a}$, $\vec{b}$, and $\vec{c}$ are all nonzero vectors, show that $\vec{c}$ bisects the angle between $\vec{a}$ and $\vec{b}$.

10. In the triangle ABC, points A_1, B_1, and C_1 are chosen on the sides BC, AC, and AB respectively in such a way that AA_1, BB_1, and CC_1 are the three (angular) bisectors of the triangle ABC. Show that, if

$$\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = \vec{0},$$

then the triangle ABC must be equilateral.