Dept. of Math. Sci., WPI
MA 1034 Analysis 4
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Homework Assignment 2

Due Tuesday, March 25, 2001

1. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors, such that $\vec{c} \neq \vec{0}$.
   (a) If $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$, does it follow that $\vec{a} = \vec{b}$? Explain.
   (b) If $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$, does it follow that $\vec{a} = \vec{b}$? Explain.
   (c) If $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$, does it follow that $\vec{a} = \vec{b}$? Explain.

2. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be four vectors. Show that
$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} \\ \vec{a} \cdot \vec{d} & \vec{b} \cdot \vec{d} \end{vmatrix}.$$

3. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors.
   (a) Show that $\vec{a} \times (\vec{b} \times \vec{c})$ is a linear combination of $\vec{b}$ and $\vec{c}$. Use the result of the previous problem to find the coefficients of this linear combination.
   (b) Show that
$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$$

4. Find the area of the triangle with vertices $P(1, -1, 2)$, $Q(3, 2, 3)$, and $R(0, 1, -1)$.

5. Find an equation for the line which both passes through $P_0(-1, 2, 3)$ and
   (a) is parallel to the line defined by $l(t) = (1, 2t, -3 + t)$.
   (b) is perpendicular to the line defined by $l(t) = (1, 2t, -3 + t)$.

6. Find the distance from the point $P(1, -1, 2)$ to the line given by $l(t) = (1 + t, 2 - 2t, 3 + t)$.

7. Consider the lines $l_1(t) = (3t + 3, t + 3, t)$ and $l_2(t) = (t - 2, t, 2t - 5)$.
   (a) Do these lines intersect?
   (b) If t represents time, and a particle travels on each line with its position determined by $l_1(t)$ and $l_2(t)$, will they ever collide?

8. Find a point where the line given by $\left(t + 1, 2t - 1, \frac{t}{3}\right)$ intersects the plane $2x - y + 3z = 6$.

9. Find the distance from the point $P(0, -1, 2)$ to the plane $2x + 3y - z = 6$.

10. Find the (acute) angle beteween the planes $2x - y = 7$ and $-x + y - 3z = 5$.