Homework Assignment 3  
Due Friday, April 18, 2003

1. Let $A, B \subseteq \mathbb{R}^n$. Prove that $(\text{int}A) \cap (\text{int}B) = \text{int}(A \cap B)$. Is the statement true if “intersection” is replaced by “union”? Explain.

2. Let $A, B \subseteq \mathbb{R}^n$. Prove that $(A' \cup B') = (A \cup B)'$. Is the statement true if “union” is replaced by “intersection”? Explain.

3. Show that the function $f(x, y) = \frac{1}{x+y}$ is continuous but not uniformly continuous on the open square $D = (0, 1) \times (0, 1)$.

In problems 4-7, find the limit, if it exists, or show that the limit does not exist.

4. 
$$\lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2}.$$  

5. 
$$\lim_{(x,y) \to (0,0)} \frac{xy \cos y}{3x^2 + y^2}.$$  

6. 
$$\lim_{(x,y) \to (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2}.$$  

7. 
$$\lim_{(x,y,z) \to (0,0,0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2}.$$  

In problems 8-10, determine the set of points where the function is continuous.

8. 
$$F(x, y) = \frac{\sin(xy)}{e^x - y^2}.$$  

9. 
$$f(x, y, z) = \frac{\sqrt{y}}{x^2 - y^2 + z^2}.$$  

10. 
$$f(x, y, z) = \arcsin \sqrt{x^2 + y^2 + z^2}.$$