

Homework Assignment 3

Due Friday, April 18, 2003

1. Let $A, B \subseteq \mathbb{R}^n$. Prove that $(\text{int}A) \cap (\text{int}B) = \text{int}(A \cap B)$. Is the statement true if “intersection” is replaced by “union”? Explain.
2. Let $A, B \subseteq \mathbb{R}^n$. Prove that $(A' \cup B') = (A \cup B)'$. Is the statement true if “union” is replaced by “intersection”? Explain.
3. Show that the function $f(x, y) = \frac{1}{x+y}$ is continuous but not uniformly continuous on the open square $D = (0, 1) \times (0, 1)$.

In problems 4-7, find the limit, if it exists, or show that the limit does not exist.

4.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}.$$

5.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy \cos y}{3x^2 + y^2}.$$

6.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2}.$$

7.

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2}.$$

In problems 8-10, determine the set of points where the function is continuous.

8.

$$F(x, y) = \frac{\sin(xy)}{e^x - y^2}$$

9.

$$f(x, y, z) = \frac{\sqrt{y}}{x^2 - y^2 + z^2}$$

10.

$$f(x, y, z) = \arcsin \sqrt{x^2 + y^2 + z^2}$$