Dept. of Math. Sci., WPI
MA 1034 Analysis 4
Bogdan Doytchinov, Term D03

Homework Assignment 4

Due Friday, April 25, 2003

1. If $f(x,y) = 3x^2y^3 - \sin x$, find f_x, f_y, f_{xx}, f_{xy}, f_{yy}, f_{xxy}, and f_{xyx}.

2. Suppose that
$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$
   (a) Use the definitions of partial derivatives to compute $f_x(0,y)$, $f_y(x,0)$, $f_{xy}(0,0)$, and $f_{yx}(0,0)$. Are the mixed partials at $(0,0)$ equal?
   (b) Compute $f_x(x,y)$ and $f_y(x,y)$ for $(x,y) \neq (0,0)$. Are the values $f_x(0,y)$ and $f_y(x,0)$ the same as the ones found in part (a)?

3. Show that $f(x,y) = \sqrt{x^2+y^2}$ is not differentiable at the origin by showing that:
   (a) there is no $\vec{m}$ as needed in Definition 11.4.1.
   (b) $f_x(0,0)$ does not exist and using part (c) of remark 11.4.2

4. Consider the function $f(x,y) = \sqrt[3]{xy}$.
   (a) Show that $f_x(0,0) = 0 = f_y(0,0)$.
   (b) Find $\nabla f(0,0)$.
   (c) Show that f is not differentiable at $(0,0)$.
   (d) Is f continuous at $(0,0)$? Explain.

5. Show that
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$
   is not differentiable at $(0,0)$.

6. Find a point (a,b) for which the function
$$f(x,y) = \begin{cases} (x-y)^2 \sin \frac{1}{x-y}, & \text{if } x \neq y, \\ 0, & \text{if } x = y. \end{cases}$$
   is differentiable at (a,b), but f_x and f_y are not continuous at (a,b).

7. If
$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0), \end{cases}$$
   show that
   (a) f is continuous at $(0,0)$.
   (b) $f_x(0,0)$ and $f_y(0,0)$ both exist.
   (c) f is not differentiable at $(0,0)$.

8. If
$$f(x,y) = \begin{cases} \frac{x^2y}{\sqrt{x^6+2y^2}}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0), \end{cases}$$
show that f is not continuous at $(0,0)$, but has a directional derivative in every direction at $(0,0)$.

9. If
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0), \end{cases}$$
show that $D_{\vec{u}}f(0,0)$ exists only if $\vec{u} = \langle 1,0 \rangle$ or $\vec{u} = \langle 0,1 \rangle$.

10. Find the unit vector in the direction in which $f(x,y) = y^2 \sin x$ increases most rapidly at the point $(0,-2)$. What is the maximum rate of change of f at $(0,-2)$?