Exercise Set 9
UTILITY MAXIMIZATION

A consumer buys two goods, coconuts and muffins. Prices of each are denoted by P_1 and P_2; the consumer's income by M. Denote the quantity of coconuts by X, that of muffins by Y.

Assume the following: P_2 = \$1, M = \$100. The budget constraint may then be written as:
$P_1X + Y = 100$

Note: In some places below, we will use Good 1 for coconuts and Good 2 for muffins. Do not be alarmed.

III. Questions

1. Select a value for P_1. Sketch the budget constraint. Sketch the indifference curve and indicate the optimal bundle on the graph. *[Obtain the optimal values from the calculator above.]*

Let P_1 = \$4/coconut.

Equation for the budget constraint:

Expenditure = Income

$P_1X + P_2Y = M$

4X + Y = 100

If we plot X on the horizontal axis, we get a negatively-sloped straight line with the following:

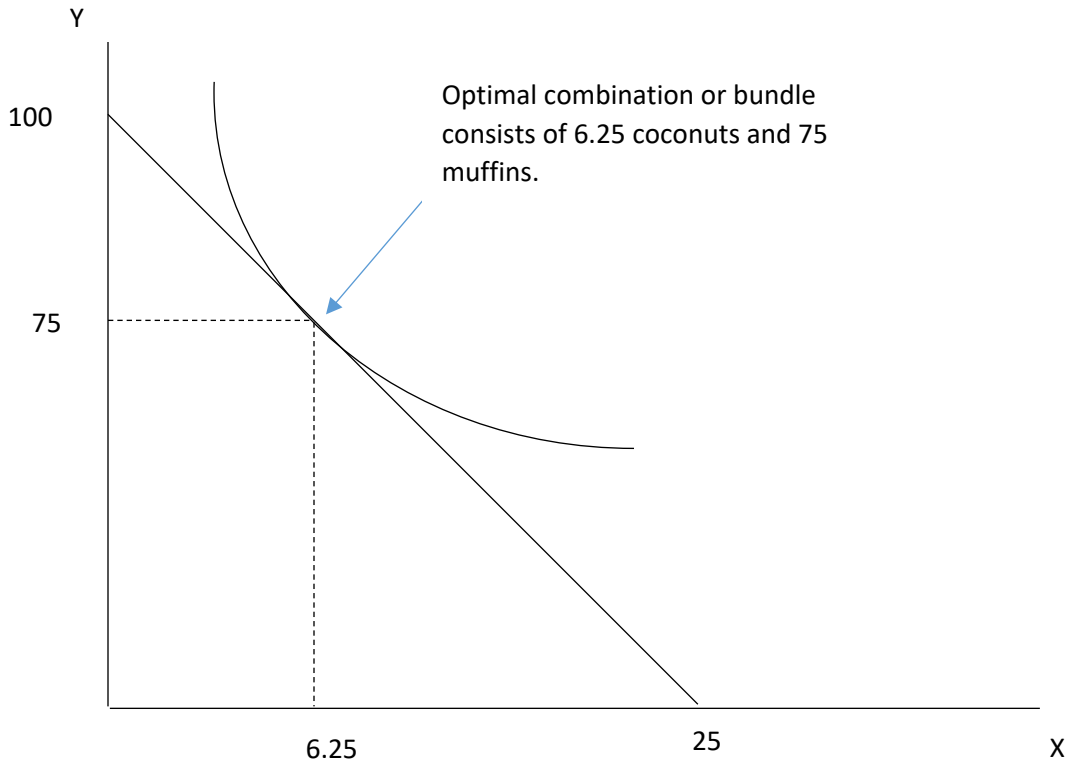
Horizontal intercept = 100/ = 25

Vertical intercept = 100/1 = 100

Slope = -100/25 = -4

Then we sketch a negatively-sloped convex indifference curve (that means "bowed in") that is tangent to the budget constraint. This will give us the **optimal bundle at the point of tangency**. Which means the consumer has attained her highest indifference curve—anything further to the right would put the combinations of goods beyond reach (of her budget).

Setting P_1 = 4 in the calculator, we obtain the optimal values as follows: X = 6.25, Y = 75.



Result: The optimal bundle is given by: $X^* = 6.25$, $Y^* = 75$.

2. Increase P_1 by 10%. The quantity demanded of coconuts will [***increase / decrease***] to _____. Sketch the new budget constraint and indifference curve. Indicate the new bundle on the graph. Has the consumer's utility increased? Explain.

   [Note a peculiar thing here: The quantity demanded of muffins has not changed. This result is unusual, and it is due to the choice of the utility function in the exercise. In a typical case, the quantity of muffins too would have changed as a result of the increase in the price of coconuts.]

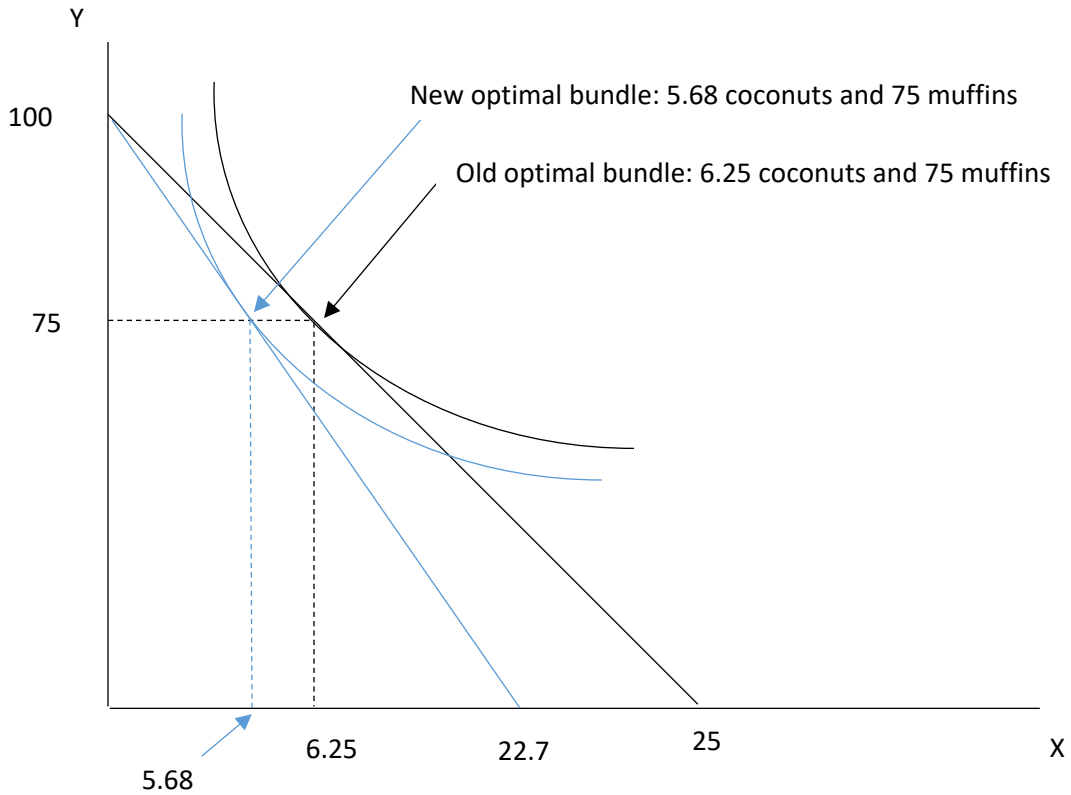
New value of $P_1 = 4.40$. (No change in P_2 and M.)

From the calculator, we get the new optimal values to be: $X = 5.68$, $Y = 75$.

The **optimal quantity demanded of coconuts has fallen** from 6.25 to 5.68, while that of muffins has stayed the same (at 75).

In the meantime, what of the budget constraint? Well, due to the increase in the price of Good 1, the constraint has **rotated inwards**. The horizontal intercept is smaller, the vertical intercept is unchanged, and the line has become steeper (higher magnitude of the slope). Yes?

Then we sketch the new indifference curve, which is tangent to the new budget constraint. Note that it lies to the left of the previous indifference curve, indicating that **the consumer's utility has fallen.**



3. Consider the case where coconuts become cheaper. Since a decrease in P_1 leads to [***more / less***] consumption of coconuts, we may conclude that the substitution effect is [***greater / less***] than the income effect. Explain.

This case is the opposite of Question 2. Here P_1 has fallen. Therefore, we can conclude that the quantity demanded of coconuts has increased while that of muffins remains the same. (You can confirm this by plugging in a suitable value for P_1 in the calculator.)

A. According to the substitution effect: When P_1 decreases, the consumer will eat ***more*** **of Good 1 (coconuts)** and *less* of the other good (muffins).

B. According to the substitution effect: When P_1 decreases, the consumer can now consume more of both goods (because purchasing power has risen). Accordingly, **amounts of both goods should rise.**

From (A) and (B): Since net quantity demanded of Good 1 (coconuts) has risen, and since both the substitution effect and income effect operate in the same direction for Good 1, **we cannot say which of the two effects is greater.**

4. Consider the effect of a decrease in P_1 on the quantity demanded of muffins. According to the substitution effect, the consumer will buy [***more / fewer***] muffins, while the income effect leads the consumer to buy [***more / fewer***] muffins. *[Since the quantity of muffins has remained unchanged in this exercise (see table above), we may conclude that the two effects have exactly offset each other. As I said earlier, highly unusual.]*

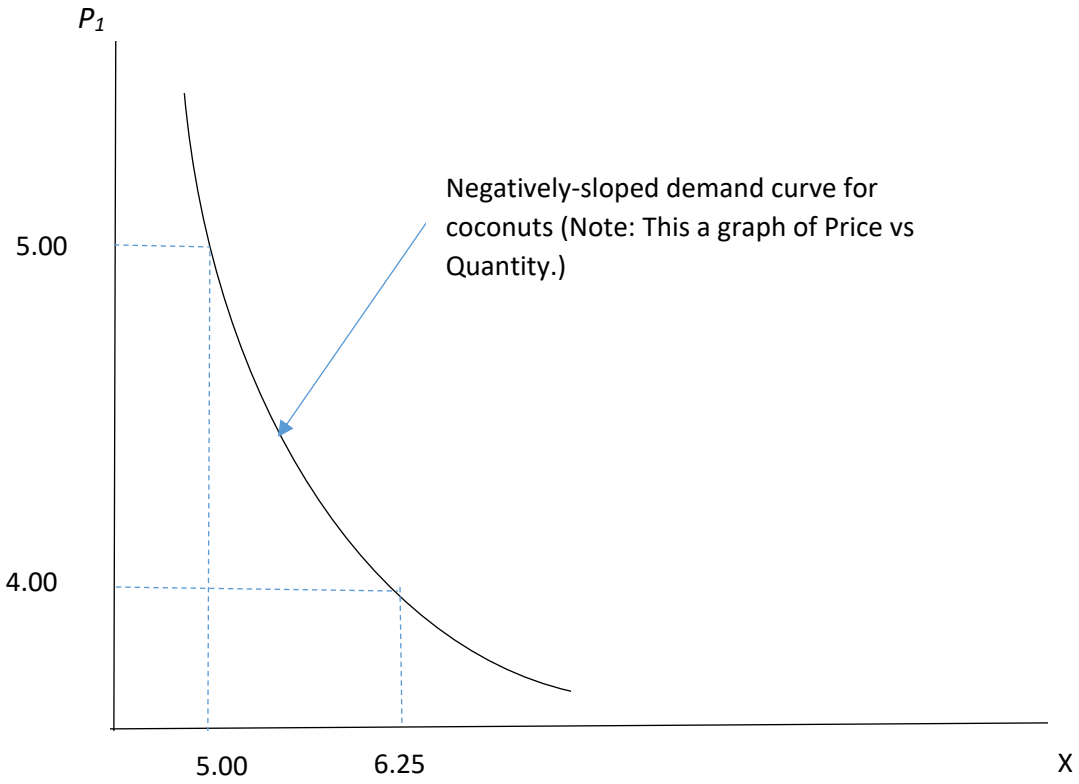
A. According to the substitution effect: When P_1 decreases, the consumer will eat *more* of Good 1 (coconuts) and ***less*** **of the other good (muffins).**

B. According to the substitution effect: When P_1 decreases, the consumer can now consume more of both goods (because purchasing power has risen). Accordingly, **amounts of both goods should rise.**

5. Change P_1 a few more times. Obtain the corresponding quantity of coconuts in each case. Sketch a demand curve for coconuts.

Here's a table with a few values of P_1 and the corresponding quantity demanded of coconuts (all taken from the calculator), followed by a sketch of the demand curve.

P_1	Quantity demanded of Good 1 (coconuts)
4.00	6.25
4.40	5.68
4.60	5.44
4.80	5.21
5.00	5.00



Result: As the price of Good 1 rises, *ceteris paribus*, the quantity demanded of Good 1 decreases.