# An Adynamical, Graphical Approach to Quantum Gravity and Unification

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- “Reconciling Spacetime and the Quantum: Relational Blockworld and the Quantum Liar Paradox,” W.M. Stuckey, Michael Silberstein & Michael Cifone, *Foundations of Physics* **38**, No. 4, 348 – 383 (2008), quant-ph/0510090

- “Why Quantum Mechanics Favors Adynamical and Acausal Interpretations such as Relational Blockworld over Backwardly Causal and Time-Symmetric Rivals,” Michael Silberstein, Michael Cifone & W.M. Stuckey, *Studies in History & Philosophy of Modern Physics* **39**, No. 4, 736 – 751 (2008)

- “Modified Regge Calculus as an Explanation of Dark Energy,” W.M. Stuckey, Timothy McDevitt & Michael Silberstein, *Classical & Quantum Gravity* **29** 055015 (2012). gr-qc/1110.3973

- “Explaining the Supernova Data without Accelerating Expansion,” W.M. Stuckey, Timothy McDevitt & Michael Silberstein. Honorable Mention in the Gravity Research Foundation 2012 Awards for Essays on Gravitation, May 2012. *International Journal of Modern Physics D* **21**, No. 11, 1242021 (2012)

- “Being, Becoming and the Undivided Universe: A Dialogue between Relational Blockworld and the Implicate Order Concerning the Unification of Relativity and Quantum Theory,” Michael Silberstein, W.M. Stuckey & Timothy McDevitt. *Foundations of Physics* **43**, No. 4, 502-532 (2013). gr-qc/1108.2261.

## 2 The Helsinki model[4]

The Helsinki model is defined by the following ingredients and principles:

- There are two kinds of primitive nodes, each the inverse of the other under reflection around the horizontal axis, and each comprising a meeting-point of three edges. If we interpret the edges as 'particle world-lines', then the nodes represent two kinds of primitive 'interaction': 'pair production' and 'pair annihilation'. (See Fig. 1)
- Each edge has one of three 'flavours', $A, B$ or $C$.
- Each node must be strictly **inhomogeneous** – i.e., comprising three edges of *different* flavours – or strictly **homogeneous** (three edges of the *same* flavour).
- Pair production and pair annihilation must alternate, when the primitive nodes are linked together.
- *Successive* homogeneous nodes are prohibited. (See Figs. 2 & 3)

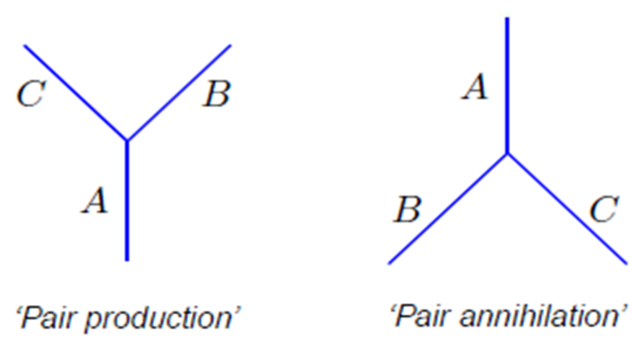



Figure 1: The two basic 'interactions'.

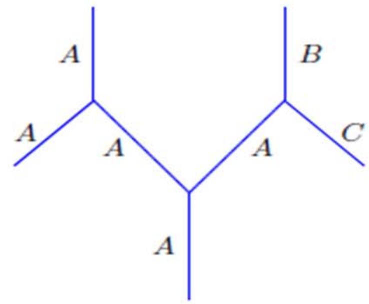



Figure 2: **Disallowed** – repeated homogeneous nodes.

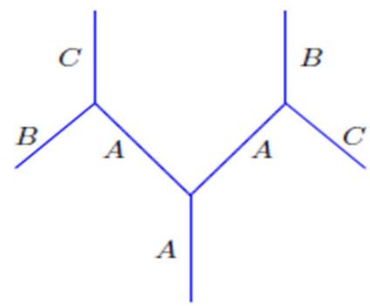



Figure 3: **Allowed** – no repeated homogeneous nodes.

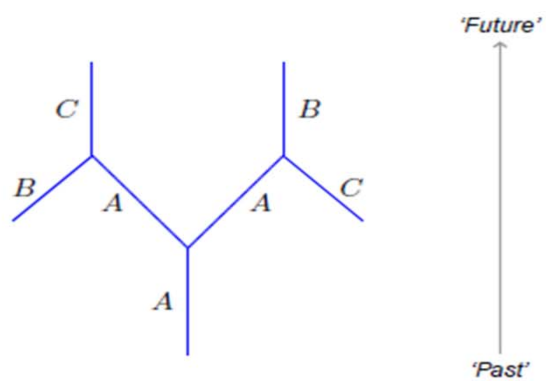
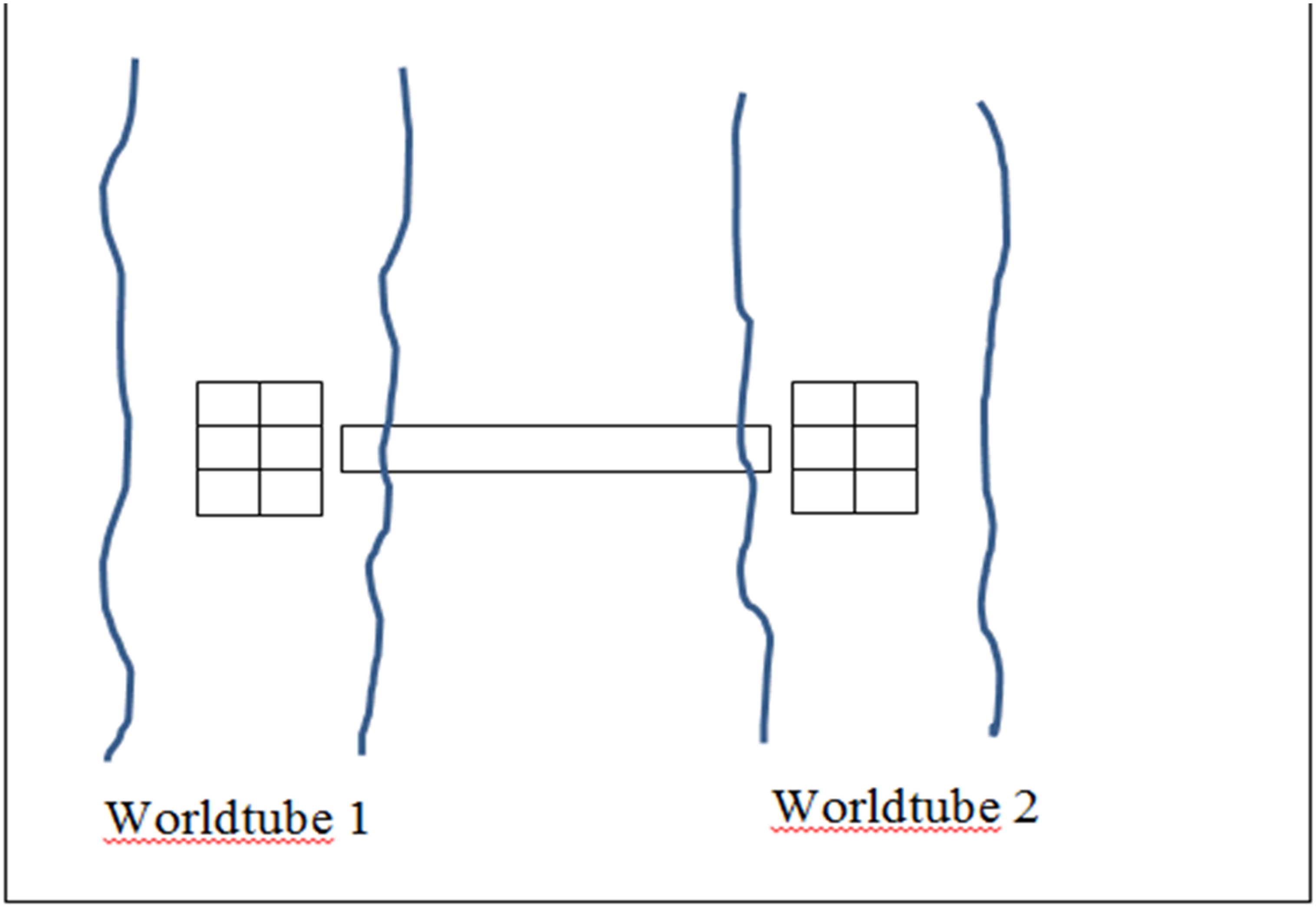


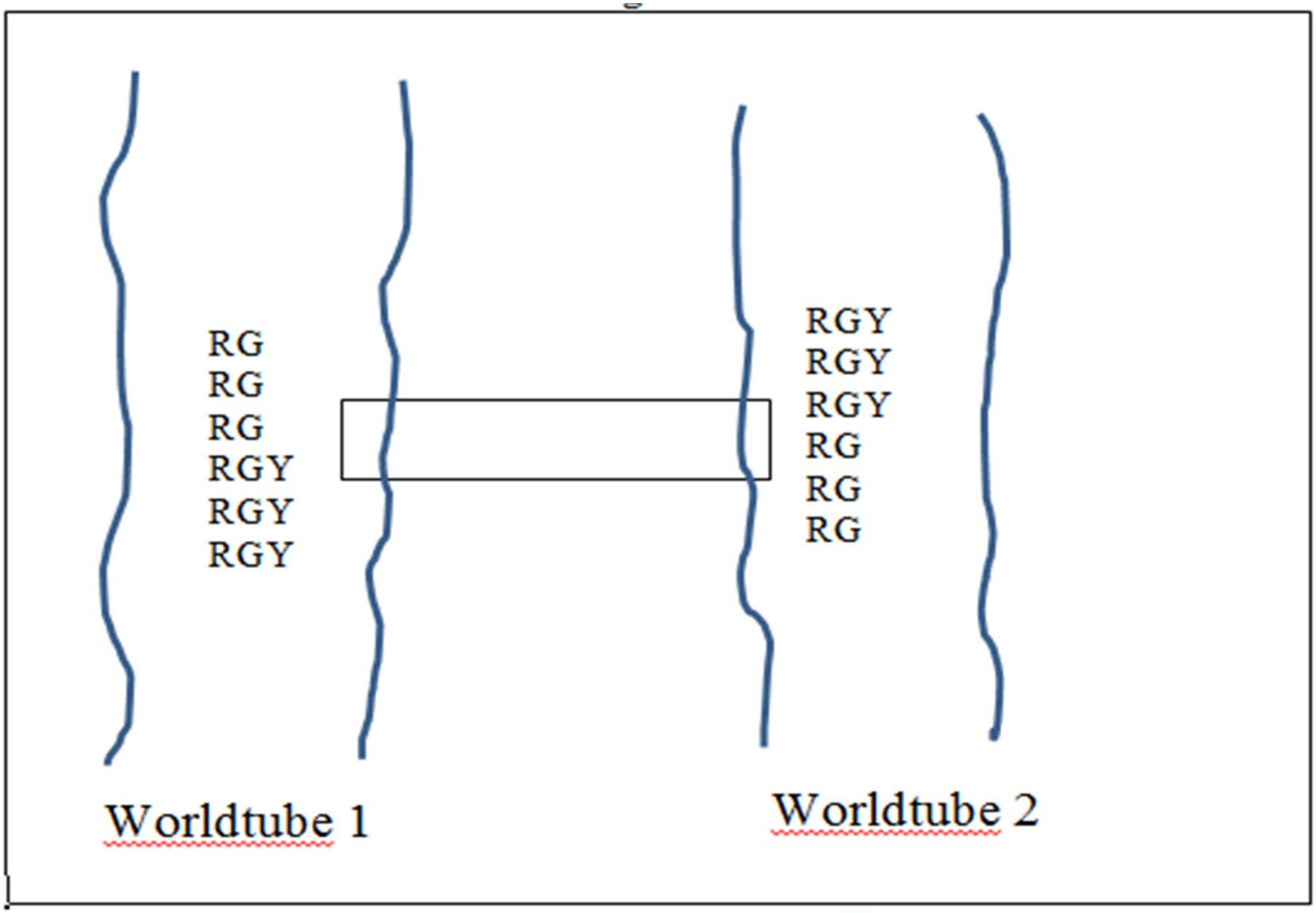

Figure 4: Adding a 'time axis'.

# Theory X per Relational Blockworld

Here we follow the possibility articulated by Wallace (p 45) that, “QFTs as a whole are to be regarded only as approximate descriptions of some as-yet-unknown deeper theory,” which he calls “theory X.” Wallace, D.: In defence of naiveté: The conceptual status of Lagrangian quantum field theory. *Synthese 151, 33-80 (2006).*




**Composition of Trans-Temporal Objects (TTOs)** – Six elements of spacetimesource are shown in each TTO's worldtube. A TTO is simply a compilation of such elements, as they account for the spatial extent of the TTO and the time-identified properties $\bar{J}$ that define the TTO. That the TTOs are themselves spatially separated means they must share elements of spacetimesource, so they must exchange $\bar{J}$ (interact). One such element is shown in this figure.




**Analogy** – The property Y is associated with the source $\bar{J}$ on the spacetimesource element shared by the worldtubes. As a result, property Y disappears from worldtube 1 (Y Source) and reappears later at worldtube 2 (Y detector). While these properties are depicted as residing in the worldtubes, they don't represent something truly intrinsic to the worldtubes, but are ultimately contextual/relational, i.e., being a Y Source only makes sense in the context of/in relation to a Y detector, and vice-versa.

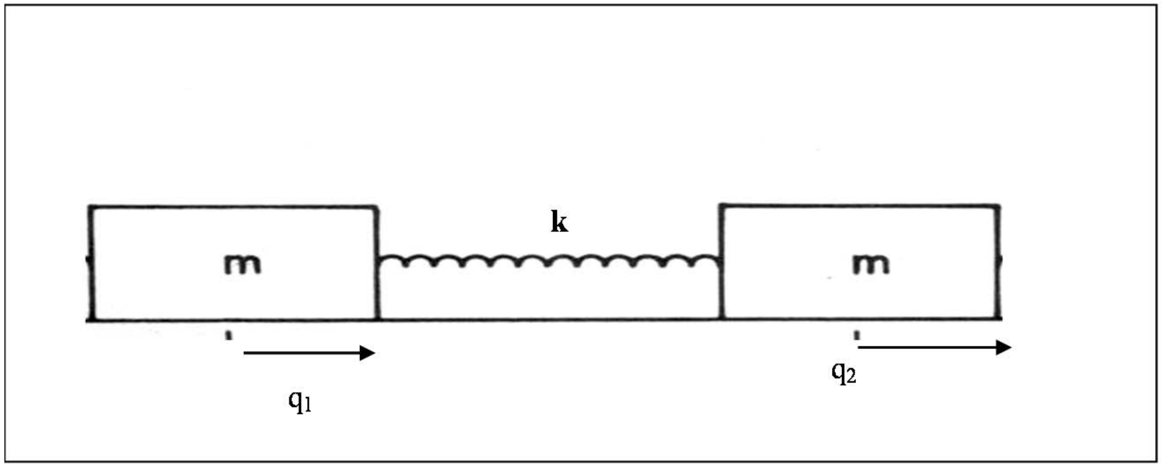
# Self Consistency Criterion

Einstein’s equations of GR are a type of SCC, i.e., you can’t solve for the metric on the LHS without the stress-energy tensor on the RHS, but you can’t produce the stress-energy tensor without a metric.

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Is there an adynamical underpinning? For EE’s it’s the boundary of a boundary principle.

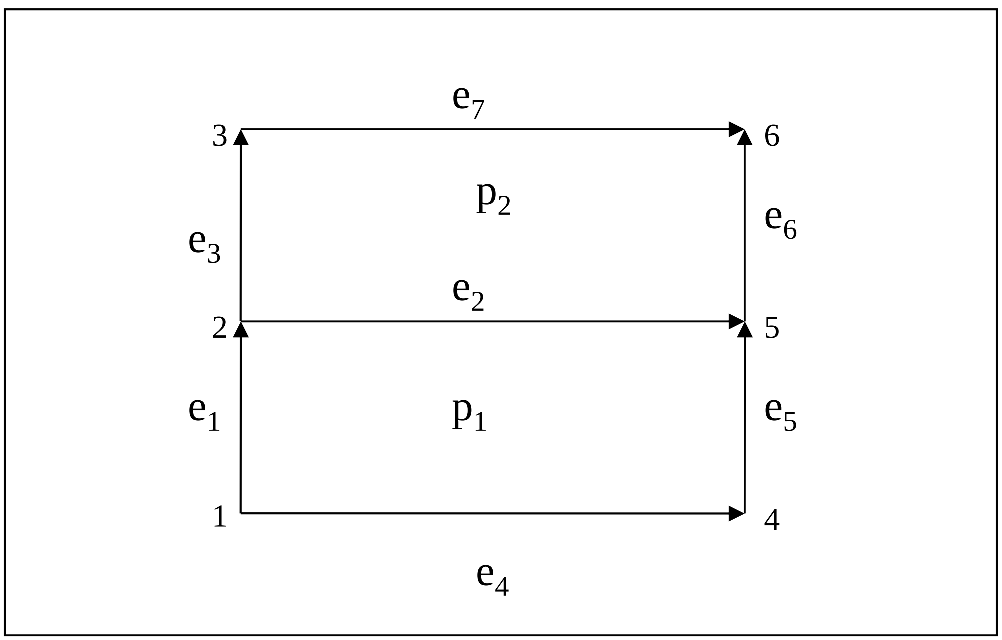
# Harmonic oscillator on a graph



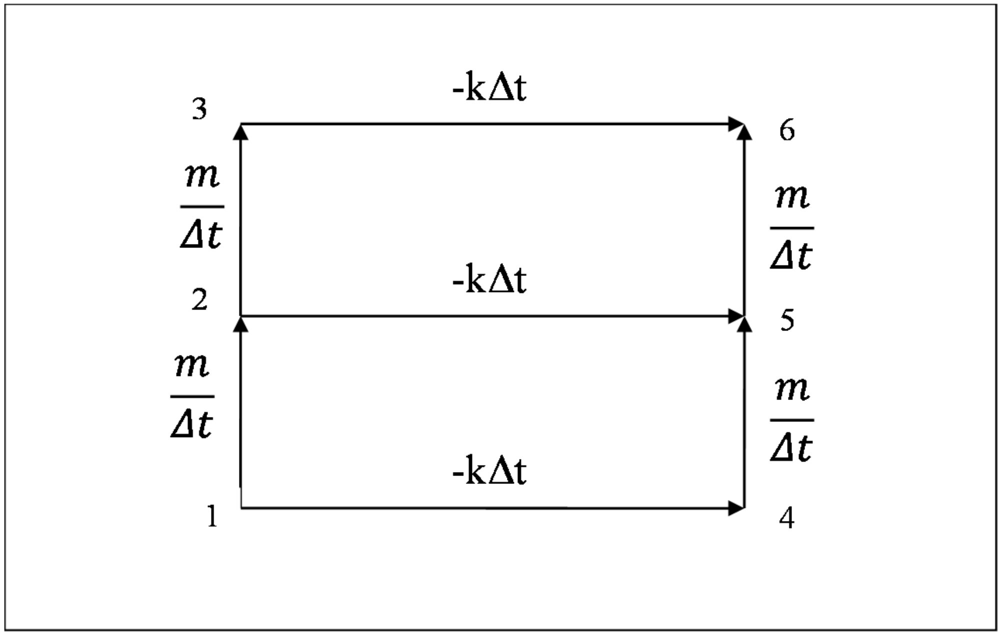

$$L = \frac{1}{2} m \dot{q}_1^2 + \frac{1}{2} m \dot{q}_2^2 - \frac{1}{2} k \left( q_1 - q_2 \right)^2$$

$$Z = \int Dq(t)\exp\left[ i\int_0^T dt\left[ \frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 - \frac{1}{2}kq_1^2 - \frac{1}{2}kq_2^2 + kq_1q_2 + J_1q_1 + J_2q_2 \right]\right]$$

$$Z = \int \ldots \int dQ_1 \ldots dQ_N \, \exp\left[ \frac{i}{2}\vec{Q}\cdot\vec{\vec{K}}\cdot\vec{Q} + i\vec{J}\cdot\vec{Q} \right]$$



$$\vec{\vec{K}} = \begin{bmatrix} \left(\frac{m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k\Delta t & 0 & 0 \\ \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k\Delta t & 0 \\ 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t} - k\Delta t\right) & 0 & 0 & k\Delta t \\ k\Delta t & 0 & 0 & \left(\frac{m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} & 0 \\ 0 & k\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} \\ 0 & 0 & k\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t} - k\Delta t\right) \end{bmatrix}$$



$$\partial_1 = \begin{bmatrix} -\sqrt{\dfrac{m}{\Delta t}} & 0 & 0 & -\sqrt{-k\Delta t} & 0 & 0 & 0 \\ \sqrt{\dfrac{m}{\Delta t}} & -\sqrt{-k\Delta t} & -\sqrt{\dfrac{m}{\Delta t}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\dfrac{m}{\Delta t}} & 0 & 0 & 0 & -\sqrt{-k\Delta t} \\ 0 & 0 & 0 & \sqrt{-k\Delta t} & -\sqrt{\dfrac{m}{\Delta t}} & 0 & 0 \\ 0 & \sqrt{-k\Delta t} & 0 & 0 & \sqrt{\dfrac{m}{\Delta t}} & -\sqrt{-k\Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{-k\Delta t} & \sqrt{-k\Delta t} \end{bmatrix}$$

$$\vec{\vec{K}} = \partial_1 \partial_1^T$$

$$a_1 = 0 \cdots a_2 = -2k\Delta t$$

$$a_3 = \frac{m}{\Delta t} \cdots a_4 = 3\frac{m}{\Delta t}$$

$$a_5 = \frac{m}{\Delta t} - 2k\Delta t \cdots a_6 = 3\frac{m}{\Delta t} - 2k\Delta t$$

Eigenvector for null space is [111111], so **SCC = divergence-free $\vec{J}$ resides in column space of $\vec{\vec{K}}$ constructed from boundary operators over graph.**

The SCC is our proposed fundamental axiom of physics, as its status in theory X is akin to Newton's laws of motion or Einstein's equations of GR. Just as Newton's second law co-defines force and mass, and Einstein's equations co-define the spacetime metic and stress-energy tensor, the SCC co-defines relations and sources at the most fundamental level of Nature. We will provide examples for the Schrödinger, Klein-Gordon, Dirac, Maxwell, and Einstein-Hilbert actions.

Now that we have explained our SCC, our choice of gauge fixing is obvious. The discrete, graphical transition amplitude is

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} dQ_1 .. dQ_N \exp\left[ i \frac{1}{2} \vec{Q} \cdot \bar{\bar{K}} \cdot \vec{Q} + i\vec{J} \cdot \vec{Q} \right]$$

with solution

$$Z = \left( \frac{(2\pi i)^N}{\det(K)} \right)^{1/2} \exp\left[ -i \frac{1}{2} \vec{J} \cdot \bar{\bar{K}}^{-1} \cdot \vec{J} \right]$$

However, $\bar{\bar{K}}^{-1}$ does not exist because $\bar{\bar{K}}$ has a non-trivial null space. This is the graphical characterization of the effect of gauge invariance on the computation of *Z*. Because we require that $\bar{J}$ reside in the column space of $\bar{\bar{K}}$, the graphical counterpart to Fadeev-Popov gauge fixing is obvious, i.e., we simply restrict our path integral to the column space of $\bar{\bar{K}}$. Nothing of physical interest lies elsewhere, so this is a natural choice. In the eigenbasis of $\bar{\bar{K}}$ with our gauge fixing we have

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} d\tilde{Q}_2 \ldots d\tilde{Q}_N \exp\left[\sum_{n=2}^{N}\left( i\frac{1}{2}\tilde{Q}_n^2 a_n + i\tilde{J}_n\tilde{Q}_n \right)\right]$$

where $\tilde{Q}_n$ are the coordinates associated with the eigenbasis of $\bar{\bar{K}}$ and $\tilde{Q}_1$ is associated with eigenvalue zero, $a_n$ is the eigenvalue of $\bar{\bar{K}}$ corresponding to $\tilde{Q}_n$, and $\tilde{J}_n$ are the components of $\bar{J}$ in the eigenbasis of $\bar{\bar{K}}$. Our gauge independent approach gives

$$Z = \left( \frac{(2\pi i)^{N-1}}{\prod_{n=2}^{N} a_n} \right)^{1/2} \prod_{n=2}^{N} \exp\left[ -i\frac{\tilde{J}_n^2}{2a_n\hbar} \right]$$

*Thus, we find that the self-consistent co-construction of space, time and divergence-free sources entails gauge invariance and gauge fixing.*

# Non-Relativistic Scalar Field on Nodes

The non-relativistic limit of the Klein-Gordon (KG) equation gives the free-particle Schrödinger equation (SE) by factoring out the rest mass contribution to the energy $E$, assuming the Newtonian form for kinetic energy, and discarding the second-order time derivative. To illustrate the first two steps, plug $\varphi = Ae^{i(px-Et)/\hbar}$ into the KG equation and obtain $\left(-E^2 + p^2c^2 + m^2c^4\right) = 0$, which tells us $E$ is the total relativistic energy. Now plug $\psi = Ae^{i(px-Et)/\hbar}$ into the free-particle SE and obtain $\frac{p^2}{2m} = E$, which tells us $E$ is only the Newtonian kinetic energy. Thus, we must factor out the rest energy of the particle, i.e., $\psi = e^{imc^2t/\hbar}\varphi$, assume the low-velocity limit of the relativistic kinetic energy, and discard the relevant term from our Lagrangian density (leading to the second-order time derivative) in going from $\varphi$ of the KG equation to $\psi$ of the free-particle SE.

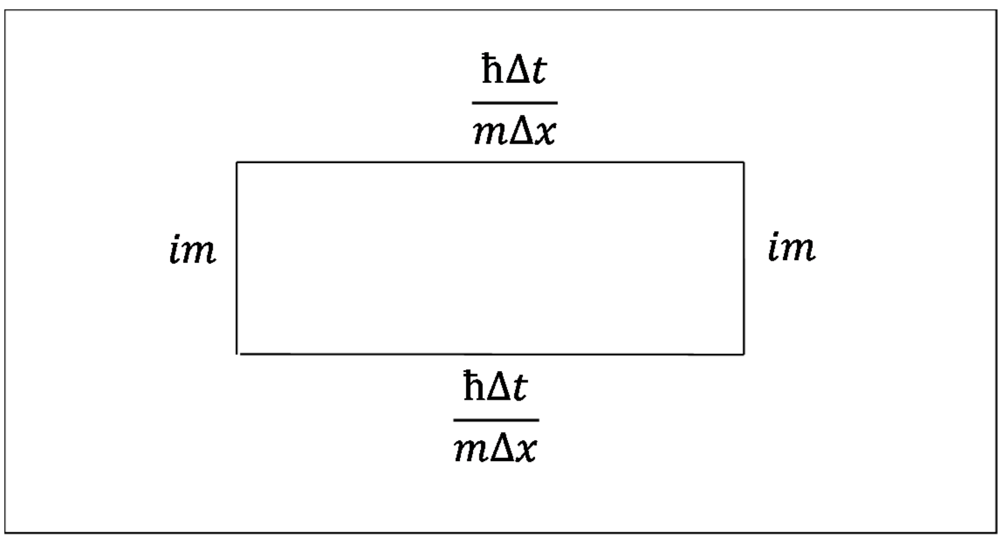
Accordingly, start with the (1+1)D KG transition amplitude

$$Z = \int D\varphi \exp\left[ i \int dxdt \left[ \frac{1}{2}\left(\frac{\partial \varphi}{\partial t}\right)^2 - \frac{c^2}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 - \frac{1}{2}\overline{m}^2 \varphi^2 + J\varphi \right]\right]$$

($\hbar = 1$ and $\overline{m} \equiv \dfrac{mc^2}{\hbar}$). Making the changes described above with $\psi = e^{\overline{m}t}\sqrt{2\overline{m}}\varphi$ gives

the non-relativistic KG transition amplitude corresponding to the free-particle SE

$$Z = \int D\varphi \exp\left[ i \int dxdt \left[ i\psi^*\left(\frac{\partial \psi}{\partial t}\right) - \frac{c^2}{2\overline{m}}\left(\frac{\partial \psi}{\partial x}\right)^2 + J\psi \right]\right]$$



$$\vec{\vec{K}} = \begin{bmatrix} \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) & im & \frac{\hbar\Delta t}{m\Delta x} & 0 \\ im & \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) & 0 & \frac{\hbar\Delta t}{m\Delta x} \\ \frac{\hbar\Delta t}{m\Delta x} & 0 & \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) & im \\ 0 & \frac{\hbar\Delta t}{m\Delta x} & im & \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) \end{bmatrix}$$

$$Z = \int \ldots \int dQ_1 \ldots dQ_N \, \exp\left[ \frac{i}{2} \vec{Q} \cdot \vec{\vec{K}} \cdot \vec{Q} + i\vec{J} \cdot \vec{Q} \right]$$

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} d\tilde{Q}_2 \ldots d\tilde{Q}_N \, \exp\left[ \sum_{n=2}^{N} \left( i \frac{1}{2} \tilde{Q}_n^2 a_n + i\tilde{J}_n \tilde{Q}_n \right) \right]$$

The eigenvalues of $\bar{\bar{K}}$ are $a_1 = 0$, $a_2 = -\dfrac{2\hbar\Delta t}{m\Delta x}$, $a_3 = -2im$, and $a_4 = a_2 + a_3$ with eigenvectors $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$, $\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}$, $\begin{bmatrix}-1\\1\\-1\\1\end{bmatrix}$, and $\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}$, respectively. The eigenvectors form the $H_4$ Hadamard matrix and the eigenvalues are consistent with this fact, i.e., 0 and -2 times the off diagonal entries of $\bar{\bar{K}}$. We choose $\bar{J}$ proportional to the unit eigenvector associated with $a_2$ (since it will give real $J^2$), which is in keeping with the SCC. Computing $Z$ per Eq (10) and using this as a propagator with a delta function Source we have

$$\psi(x,t) \propto \exp\left[\frac{iJ_o^2}{2\dfrac{2\hbar t}{mx}\hbar}\right]$$

where $J_o$ is the magnitude of $\bar{J}$ ($\Delta t \to t$ and $\Delta x \to x$ for notational simplicity).

The corresponding QM propagator is obtained via the path integral with action

$$S = \int \frac{1}{2} m \left( \frac{dx}{dt} \right)^2 dt$$

which gives

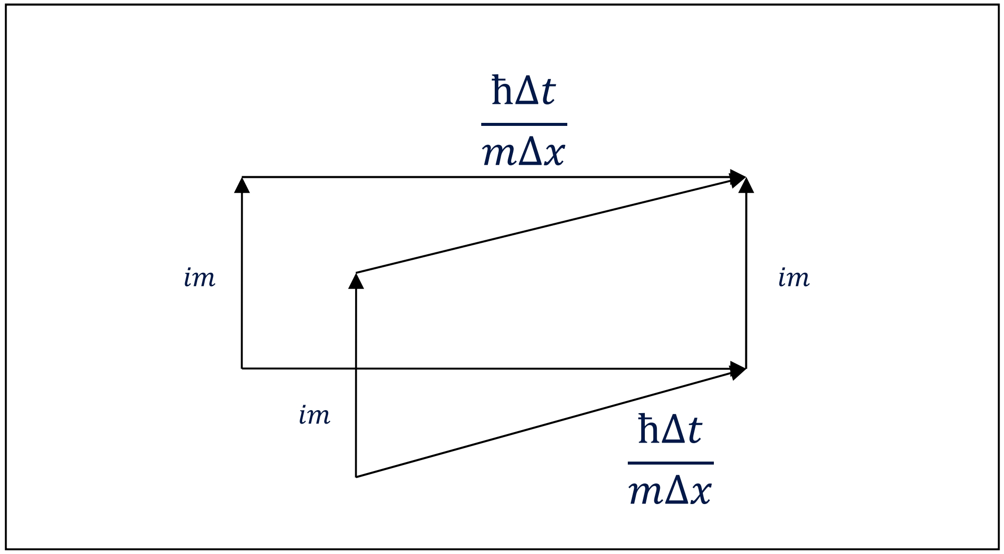
$$\psi(x,t) = A\sqrt{\frac{m}{2\pi\hbar i t}} \exp\left[\frac{imx^2}{2\hbar t}\right] \propto \exp\left[\frac{imx^2}{2\hbar t}\right]$$

with delta function Source $\psi(x,0) = A\delta(x)$. In this view, a particle of mass $m$ is moving through space from Source to detector, so we call this a "mediated" view.

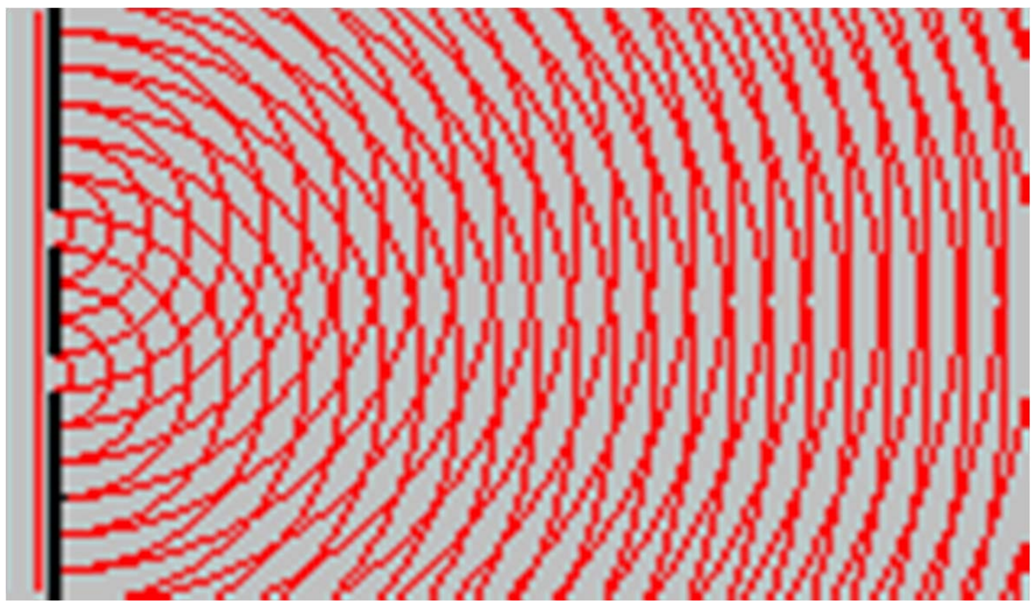
Comparing the exponents of theory X and QM we have $J_o^2 = 2\hbar x$ . Thus, in GR-like fashion, we obtain a self-consistency relationship between source and space resulting from our fundamental axiom of physics.

Both solutions are oscillatory, so it is easy to see how both results lead to twin-slit interference. However, the results are quite different conceptually. Theory X was obtained in spatiotemporally holistic fashion and the view of how its amplitudes are combined is shown at the top of the next slide. By contrast, QM's solution was obtained dynamically and the view of how its amplitudes are combined is shown at the bottom of the next slide. This illustrates nicely that per theory X the interference pattern of the twin-slit experiment does not entail "quantum entities" moving through space as a function of time to "cause" detector events. Rather, interference is understood adynamically via 'competition' between fundamental elements of spacetimesource.

Again, in our view, physics is concerned with explaining the relative spatiotemporal locations of TTOs and physics currently says TTOs are composed of smaller TTOs, i.e., smaller subsets of trans-temporally identified properties (fundamental particles). We propose a more fundamental decomposition of TTOs in terms of spacetimesource elements. Accordingly, quantum physics is telling us something very important about the composition of TTOs, i.e., their properties combine via interference at the level of spacetimesouce elements.

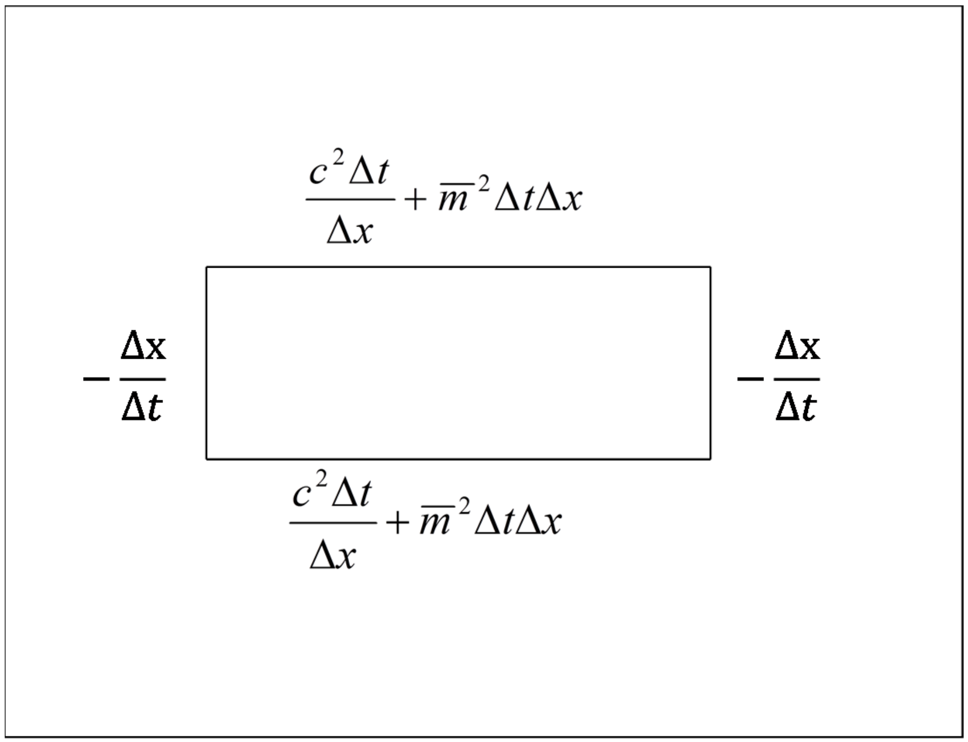


$\frac{\hbar\Delta t}{m\Delta x}$
$im$
$im$
$im$
$\frac{\hbar\Delta t}{m\Delta x}$

# Scalar Field on Nodes

$$Z = \int D\varphi \exp\left[ i\int dxdt \left[ \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{c^2}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - \frac{1}{2}\overline{m}^2\varphi^2 + J\varphi \right]\right]$$



$$\vec{\vec{K}} = \begin{bmatrix} \left( \frac{\Delta x}{\Delta t} - \frac{c^2 \Delta t}{\Delta x} - \overline{m}^2 \Delta t \Delta x \right) & -\frac{\Delta x}{\Delta t} & \frac{c^2 \Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x & 0 \\ -\frac{\Delta x}{\Delta t} & \left( \frac{\Delta x}{\Delta t} - \frac{c^2 \Delta t}{\Delta x} - \overline{m}^2 \Delta t \Delta x \right) & 0 & \frac{c^2 \Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x \\ \frac{c^2 \Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x & 0 & \left( \frac{\Delta x}{\Delta t} - \frac{c^2 \Delta t}{\Delta x} - \overline{m}^2 \Delta t \Delta x \right) & -\frac{\Delta x}{\Delta t} \\ 0 & \frac{c^2 \Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x & -\frac{\Delta x}{\Delta t} & \left( \frac{\Delta x}{\Delta t} - \frac{c^2 \Delta t}{\Delta x} - \overline{m}^2 \Delta t \Delta x \right) \end{bmatrix}$$

The eigenvalues of $\bar{\bar{K}}$ are $a_1 = 0$, $a_2 = -2\left(\frac{c^2 \Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x\right)$, $a_3 = \frac{2\Delta x}{\Delta t}$, and $a_4 = a_2 + a_3$

with the same eigenvectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$, respectively, again we have the

$H_4$ Hadamard matrix with eigenvalues of 0 and -2 times the off-diagonal entries of $\bar{\bar{K}}$. As with the non-relativistic case, we choose $\bar{J}$ proportional to the unit eigenvector associated with $a_2$. In this case, our Z gives (dropping $\Delta$)

$$\varphi(x,t) \propto \exp\left[\frac{iJ_o^2}{4\left(\frac{c^2 t}{x} + \overline{m}^2 tx\right)\hbar}\right]$$

Again, we wish to compare with the mediated counterpart, so we compare with the two-point correlation function for the free scalar field

$$G(x,t) \propto e^{i\frac{px}{\hbar} - i\frac{Et}{\hbar}}$$

Comparing the exponents we obtain $J_o^2 = 4\left(\frac{c^2 t}{x} + \overline{m}^2 tx\right)(px - Et)$. Here the SCC leads to the self-consistent relationship between source, time, space, mass, momentum, and energy. To see how this reduces to our non-relativistic result, we first reintroduce the scaling factor $\sqrt{\overline{m}}$ so that

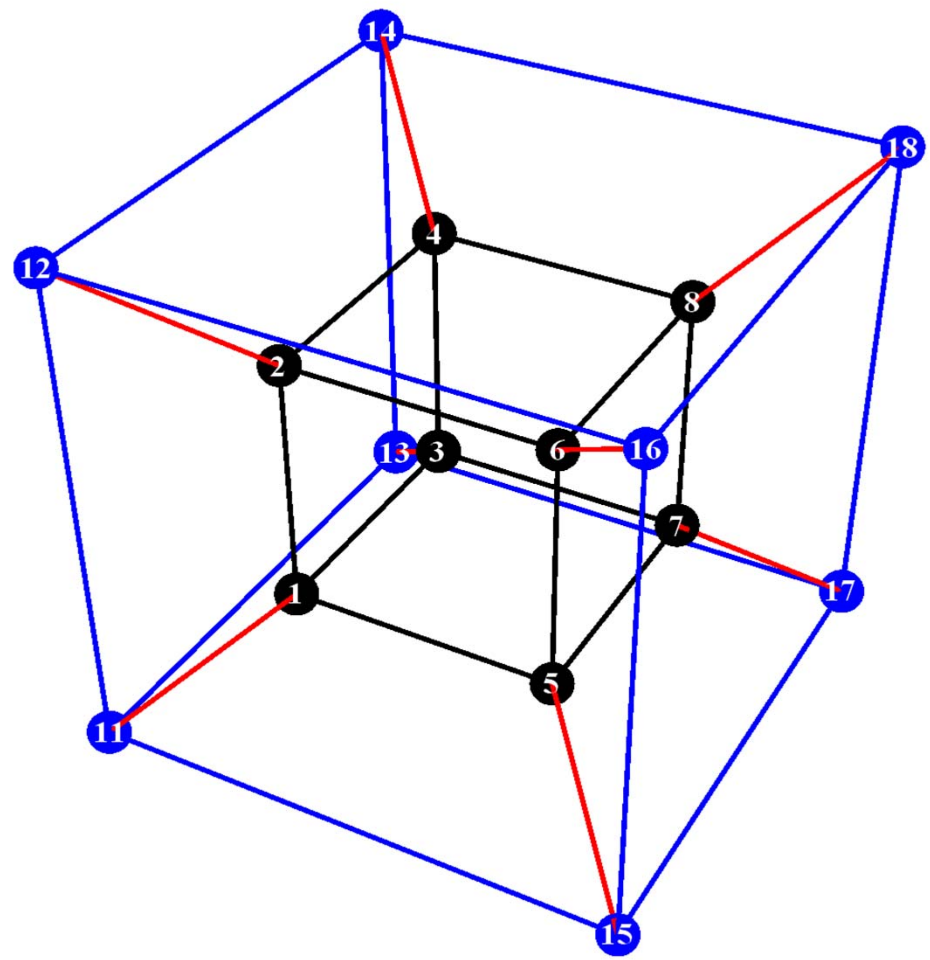
$a_2 = -2\left(\frac{c^2 \Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x\right) \rightarrow -2\left(\frac{c^2 \Delta t}{\overline{m}\Delta x} + \overline{m}\Delta t \Delta x\right) = -2\left(\frac{\hbar \Delta t}{m\Delta x} + \frac{mc^2 \Delta t \Delta x}{\hbar}\right)$. Then our non-relativistic result follows from $\frac{\hbar t}{mx} + \overline{m}tx \rightarrow \frac{\hbar t}{mx}$, $p = m\frac{x}{t}$ and $E = \frac{1}{2}m\left(\frac{x}{t}\right)^2$, as we would expect.

# Vector Field on Nodes

We apply this approach to vector fields on nodes and note that the KG operator for scalar fields is the square of the Dirac operator for vector fields, i.e., $(-i\gamma^{\mu}\partial_{\mu}-m)(i\gamma^{\mu}\partial_{\mu}-m)=(\partial^{2}+m^{2})$. In order to construct $\bar{\bar{K}}$ for the Dirac operator on the hypercube we have the following link weights on $t$, $x$, $y$, and $z$ links respectively:

$$T=\begin{bmatrix} \frac{i}{t}-m & 0 & 0 & 0 \\ 0 & \frac{i}{t} & 0 & 0 \\ 0 & 0 & \frac{i}{t} & 0 \\ 0 & 0 & 0 & \frac{i}{t} \end{bmatrix} \qquad X=\begin{bmatrix} 0 & 0 & 0 & \frac{i}{x} \\ 0 & -m & \frac{i}{x} & 0 \\ 0 & \frac{i}{x} & 0 & 0 \\ \frac{i}{x} & 0 & 0 & 0 \end{bmatrix}$$

$$Y=\begin{bmatrix} 0 & 0 & 0 & \frac{1}{y} \\ 0 & 0 & -\frac{1}{y} & 0 \\ 0 & \frac{1}{y} & m & 0 \\ -\frac{1}{y} & 0 & 0 & 0 \end{bmatrix} \qquad Z=\begin{bmatrix} 0 & 0 & \frac{i}{z} & 0 \\ 0 & 0 & 0 & -\frac{i}{z} \\ \frac{i}{z} & 0 & 0 & 0 \\ 0 & -\frac{i}{z} & 0 & m \end{bmatrix}$$


14
18
4
12
8
2
13
3
6
16
7
17
1
5
11
15

Then the 64 x 64 matrix $\bar{\bar{K}}$ is simply given by:

$$\bar{\bar{K}} = \begin{bmatrix} (-T-X-Y-Z) & Z & Y & 0 & X & \dots \\ \vdots & & & & & \end{bmatrix}$$

This has the same form as $\bar{\bar{K}}$ for the Schrödinger and the KG actions. That is, reading across the rows for each node one simply has a collection of the link weights relating the nodes which are connected. Thus, we can understand how $\bar{\bar{K}}$ instantiates graphical relationalism and divergence-free $\bar{J}$ per the SCC as follows.

Each row of $\bar{\bar{K}}$ is a vector constructed relationally via the connectivity of some graphical element, i.e., nodes connected by links, links connected by plaquettes, or plaquettes connected by cubes. Since each vector is relationally defined, its elements sum to zero, which means $[111\dots]$ is a null eigenvector of $\bar{\bar{K}}$. Thus, the determinant of $\bar{\bar{K}}$ is zero, so the set of row vectors is not linearly independent. That some subset of the vectors is determined by its complement follows from having the graphical set relationally defined. Therefore, divergence-free $\bar{J}$ follows from relationally defined $\bar{\bar{K}}$ as a consequence of our SCC.

To study the eigenstructure, we point out that $\bar{\bar{K}}$ is in nested form. $\bar{\bar{K}}_{block} = \begin{bmatrix} A & TI \\ TI & A \end{bmatrix}$

where $TI$ is the 8x8 identity matrix $I$ times $T$ and $A$ is the 8x8 matrix $A = \begin{bmatrix} B & XI \\ XI & B \end{bmatrix}$.

Continuing the nesting we have $B = \begin{bmatrix} C & YI \\ YI & C \end{bmatrix}$ where $C = \begin{bmatrix} D & ZI \\ ZI & D \end{bmatrix}$ and

$D = [-T - X - Y - Z]$. The eigenvalue problem for $\bar{\bar{K}}$ then takes a nested form in terms of Hadamard matrices $H_1$, $H_2$, $H_4$, $H_8$, and $H_{16}$ as follows. $DH_1 = H_1[-T - X - Y - Z]$ where $H_1 = [1]$. $C\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -T - X - Y & 0 \\ 0 & -T - X - Y - 2Z \end{bmatrix}$ where $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

$BH_4 = H_4 diag[-T - X, -T - X - 2Z, -T - X - 2Y, -T - X - 2Y - 2Z]$ where

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

$$AH_8 = H_8 diag[-T, -T-2Z, -T-2Y-2Z, -T-2X, -T-2X-2Z, -T-2X-2Y, -T-2X-2Y-2Z]$$

where $H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} = H_2 \otimes H_4$. Thus, $\bar{\bar{K}}_{block} H_{16} = H_{16} diag[vector]$ where

$H_{16} = H_2 \otimes H_8$ and

$$vector = -2\begin{bmatrix} 0 \\ Z \\ Y \\ Y+Z \\ X \\ X+Z \\ X+Y \\ X+Y+Z \\ T \\ T+Z \\ T+Y \\ T+Y+Z \\ T+X \\ T+X+Z, \\ T+X+Y \\ T+X+Y+Z \end{bmatrix}$$

Finally, the eigenvalue problems for each of the 4x4 matrices in *vector* are solved and the eigenvectors are located in a 64x64 matrix built from $H_{16}$. So, for example, the first column of $H_{16}$ is [111…] and the four-dimensional null space is be spanned by [1,0,0,0], [0,1,0,0], [0,0,1,0] and [0,0,0,1], so the first four columns of the eigenbasis matrix for $\bar{\bar{K}}$ are (column entries top to bottom read left to right here):

```
{{0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0,
  0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0,
  0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0},
 {0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,
  0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1,
  0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0}}
```

$\bar{J}$ being orthogonal to each of these vectors simply means that the global sum over each spacetime component of $\bar{J}$ at each node gives zero, as required for vector addition over all 16 nodes.

# Scalar Field on Links

We now apply this approach to gauge fields for the exchange of energy via photons. In order to model the construct of action for the exchange of energy via photons, we use the Maxwell Lagrangian density $L$ for free electromagnetic radiation

$$L = -\frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta}$$

with the field strength tensor given by

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha = \left[ \frac{\left(A_\beta(n+\hat{\alpha}) - A_\beta(n)\right)}{\ell_\alpha} - \frac{\left(A_\alpha(n+\hat{\beta}) - A_\alpha(n)\right)}{\ell_\beta} \right]$$

on the graph where $n$ is the node number, $\ell_i$ the lattice spacing in the $i^{th}$ direction, and $\hat{\alpha}$ and $\hat{\beta}$ are displacements to adjoining nodes in those directions.

Applying this to the (1+1)D case $\bar{\bar{K}}$ is

$$\begin{matrix} \frac{1}{x^2} & -\frac{1}{x^2} & -\frac{1}{tx} & \frac{1}{tx} \\ -\frac{1}{x^2} & \frac{1}{x^2} & \frac{1}{tx} & -\frac{1}{tx} \\ -\frac{1}{tx} & \frac{1}{tx} & \frac{1}{t^2} & -\frac{1}{t^2} \\ \frac{1}{tx} & -\frac{1}{tx} & -\frac{1}{t^2} & \frac{1}{t^2} \end{matrix}$$

where we have ignored overall factors $\frac{-1}{4\mu_o}$ and the volume of the element, and $c = 1$. The eigenvalues are 0, 0, 0, $2\left(\frac{1}{x^2}+\frac{1}{t^2}\right)$. The dimensionality of the column space represents the degrees of freedom available with local conservation of $\bar{J}$. That is, specifying $\bar{J}$ on just one link dictates the other three values per conservation of $\bar{J}$ on the links at each node.

On the cube $\vec{\vec{K}}$ is

$$
\begin{array}{ccccccccc}
-\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{t^2} & -\frac{1}{t\,x} & \frac{1}{t\,x} & \frac{1}{y^2} & 0 & 0 & 0 & \frac{1}{x\,y} \\
\frac{1}{t^2} & -\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{t\,x} & -\frac{1}{t\,x} & 0 & \frac{1}{y^2} & 0 & 0 & 0 \\
-\frac{1}{t\,x} & \frac{1}{t\,x} & -\frac{1}{x^2}-\frac{1}{y^2} & \frac{1}{x^2} & 0 & 0 & \frac{1}{y^2} & 0 & -\frac{1}{t\,y} \\
\frac{1}{t\,x} & -\frac{1}{t\,x} & \frac{1}{x^2} & -\frac{1}{x^2}-\frac{1}{y^2} & 0 & 0 & 0 & \frac{1}{y^2} & 0 \\
\frac{1}{y^2} & 0 & 0 & 0 & -\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{t^2} & -\frac{1}{t\,x} & \frac{1}{t\,x} & -\frac{1}{x\,y} \\
0 & \frac{1}{y^2} & 0 & 0 & \frac{1}{t^2} & -\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{t\,x} & -\frac{1}{t\,x} & 0 \\
0 & 0 & \frac{1}{y^2} & 0 & -\frac{1}{t\,x} & \frac{1}{t\,x} & -\frac{1}{x^2}-\frac{1}{y^2} & \frac{1}{x^2} & \frac{1}{t\,y} \\
0 & 0 & 0 & \frac{1}{y^2} & \frac{1}{t\,x} & -\frac{1}{t\,x} & \frac{1}{x^2} & -\frac{1}{x^2}-\frac{1}{y^2} & 0 \\
\frac{1}{x\,y} & 0 & -\frac{1}{t\,y} & 0 & -\frac{1}{x\,y} & 0 & \frac{1}{t\,y} & 0 & -\frac{1}{t^2} \\
0 & \frac{1}{x\,y} & \frac{1}{t\,y} & 0 & 0 & -\frac{1}{x\,y} & -\frac{1}{t\,y} & 0 & \frac{1}{t^2} \\
-\frac{1}{x\,y} & 0 & 0 & -\frac{1}{t\,y} & \frac{1}{x\,y} & 0 & 0 & \frac{1}{t\,y} & \frac{1}{x^2} \\
0 & -\frac{1}{x\,y} & 0 & \frac{1}{t\,y} & 0 & \frac{1}{x\,y} & 0 & -\frac{1}{t\,y} & 0
\end{array}
$$

The eigenvalues for $\bar{\bar{K}}$ are

$$\left\{0,0,0,0,0,0,0,-\frac{2(t^2+x^2)}{t^2x^2},-\frac{2(t^2+y^2)}{t^2y^2},-\frac{2(x^2+y^2)}{x^2y^2},-\frac{2(t^2x^2+t^2y^2+x^2y^2)}{t^2x^2y^2},-\frac{2(t^2x^2+t^2y^2+x^2y^2)}{t^2x^2y^2}\right\}$$

Again, the dimensionality of the column space (five) represents the degrees of freedom available with local conservation of $\bar{J}$. That is, specifying $\bar{J}$ on the four links of one face (front, say) gives $\bar{J}$ on the links connecting the front face to the back face by local conservation. Then specifying $\bar{J}$ on just one link of the back face specifies the remaining links by local conservation.

$\bar{\bar{K}}$ for the hypercube is too large to display here, but its eigenvalues are

$$\left\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{-\frac{2}{t^2} - \frac{2}{x^2}\right\},\right.$$
$$\left\{-\frac{2}{t^2} - \frac{2}{y^2}\right\}, \left\{\frac{2}{x^2} + \frac{2}{y^2}\right\}, \left\{\frac{2\left(t^2 - x^2\right)}{t^2 x^2} + \frac{2}{y^2}\right\}, \left\{-\frac{2\left(t^2 + x^2\right)}{t^2 x^2} - \frac{2}{y^2}\right\},$$
$$\left\{-\frac{2}{t^2} - \frac{2}{z^2}\right\}, \left\{\frac{2}{x^2} + \frac{2}{z^2}\right\}, \left\{\frac{2\left(t^2 - x^2\right)}{t^2 x^2} + \frac{2}{z^2}\right\}, \left\{-\frac{2\left(t^2 + x^2\right)}{t^2 x^2} - \frac{2}{z^2}\right\}, \left\{\frac{2}{y^2} + \frac{2}{z^2}\right\},$$
$$\left\{\frac{2\left(t^2 - y^2\right)}{t^2 y^2} + \frac{2}{z^2}\right\}, \left\{-\frac{2\left(t^2 + y^2\right)}{t^2 y^2} - \frac{2}{z^2}\right\}, \left\{\frac{2\left(x^2 + y^2\right)}{x^2 y^2} + \frac{2}{z^2}, \frac{2\left(x^2 + y^2\right)}{x^2 y^2} + \frac{2}{z^2}\right\},$$
$$\left.\left\{\frac{2\left(t^2 x^2 + t^2 y^2 - x^2 y^2\right)}{t^2 x^2 y^2} + \frac{2}{z^2}, \frac{2\left(t^2 x^2 + t^2 y^2 - x^2 y^2\right)}{t^2 x^2 y^2} + \frac{2}{z^2}\right\}, \left\{-\frac{2\left(t^2 x^2 + t^2 y^2 + x^2 y^2\right)}{t^2 x^2 y^2} - \frac{2}{z^2}\right\}\right\}$$

Again, the dimensionality of the column space (17) represents the degrees of freedom available with local conservation of $\bar{J}$. If we specify $\bar{J}$ on all 12 links of the "inner" cube, all the time-like links connecting the "inner" cube to the "outer" cube are determined by local conservation. Then if you specify the 4 link values on one face of the "outer" cube, local conservation leaves only one free link to specify on the opposite face, giving $12 + 4 + 1 = 17$.

# Scalar Field on Plaquettes

This is linearized GR, i.e., the harmonic terms only. We have for the Einstein-Hilbert Lagrangian density

$$L = -\partial_\lambda h_{\alpha\beta} \partial^\lambda h^{\alpha\beta} + 2\partial_\lambda h_{\alpha\beta} \partial^\beta h^{\alpha\lambda}$$

omitting trace terms gauge equivalent to $2\partial_\alpha h^\alpha_\mu$ which would be used for juxtaposed graphical elements, i.e., a more complex arrangement. To discretize this on the hypercube we first label our scalar field on each plaquette according to its span. For example, the front face of the "inner" cube is spanned by $x$ and $z$, so it's labeled $h_{13}$. Of course, there are three other such plaquettes, one displaced from the front towards the back (in $y$) of the "inner" cube, one displaced in $t$ to the front of the "outer" cube, and one displaced in $t$ and $y$ to the back of the "outer" cube. There are six fields ($h_{01}$, $h_{02}$, $h_{03}$, $h_{12}$, $h_{13}$, $h_{23}$) which generate such a quadruple, accounting for all 24 plaquettes of the hypercube. Likewise, for the cube we have ($h_{01}$, $h_{02}$, $h_{12}$) and their pairing partners giving us the six plaquettes.

We see that the first term of $S$ is just the sum of the squares of the gradients formed in each set of $h_{\alpha\beta}$ values, e.g.,

$$\left(\frac{h_{13}(back-in)}{y}-\frac{h_{13}(front-in)}{y}\right)^2+\left(\frac{h_{13}(back-out)}{y}-\frac{h_{13}(front-out)}{y}\right)^2+$$
$$\left(\frac{h_{13}(back-out)}{ct}-\frac{h_{13}(back-in)}{ct}\right)^2+\left(\frac{h_{13}(front-out)}{ct}-\frac{h_{13}(front-in)}{ct}\right)^2$$

for $h_{13}$ where "in" stands for "inner" cube and "out" stands for "outer" cube. The second term of $S$ is formed by mixing gradients, just as with the photon field. For example, we would have terms like $(\partial_0 h_{12})(\partial_2 h_{10})$ which on the lattice would have forms such as

$$\left(\frac{h_{12}(front-out)}{t}-\frac{h_{12}(front-in)}{t}\right)\left(\frac{h_{10}(back-in)}{y}-\frac{h_{10}(front-in)}{y}\right)$$

Using these conventions on the cube (again, ignoring overall scaling factors and letting $c = 1$), $\bar{\bar{K}}$ is

$$\begin{matrix} \frac{1}{t^2} & -\frac{1}{t^2} & -\frac{1}{ty} & \frac{1}{ty} & -\frac{1}{tx} & \frac{1}{tx} \\ -\frac{1}{t^2} & \frac{1}{t^2} & \frac{1}{ty} & -\frac{1}{ty} & \frac{1}{tx} & -\frac{1}{tx} \\ -\frac{1}{ty} & \frac{1}{ty} & \frac{1}{x^2} & -\frac{1}{x^2} & -\frac{1}{xy} & \frac{1}{xy} \\ \frac{1}{ty} & -\frac{1}{ty} & -\frac{1}{x^2} & \frac{1}{x^2} & \frac{1}{xy} & -\frac{1}{xy} \\ -\frac{1}{tx} & \frac{1}{tx} & -\frac{1}{xy} & \frac{1}{xy} & \frac{1}{y^2} & -\frac{1}{y^2} \\ \frac{1}{tx} & -\frac{1}{tx} & \frac{1}{xy} & -\frac{1}{xy} & -\frac{1}{y^2} & \frac{1}{y^2} \end{matrix}$$

which looks much like $\bar{\bar{K}}$ for the (1+1)D scalar field on links.

The eigenvalues of $\bar{\bar{K}}$ are

$$\{0,0,0,2(\frac{1}{x^2}+\frac{1}{y^2}),\frac{xy-\sqrt{x^2y^2+4t^2(x^2+y^2)}}{t^2xy},\frac{xy+\sqrt{x^2y^2+4t^2(x^2+y^2)}}{t^2xy}\}$$

and a basis for the null space is

$$\{\{0,0,0,0,1,1\},\{0,0,1,1,0,0\},\{1,1,0,0,0,0\}\}$$

which represents conservation of $\bar{J}$ among each pair of plaquettes associated with $(h_{01}, h_{02}, h_{12})$. [Of course, the rows of $\bar{\bar{K}}$ sum to zero so, as always, [111…] is a null eigenvector meaning we have global conservation of $\bar{J}$.]

$\bar{\bar{K}}$ for the hypercube is too large to display here, but one null eigenbasis is

{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0},
{0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0},
{0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}}

$\bar{J}$ orthogonal to each of these null eigenvectors means it is conserved across each set of four plaquettes associated with ($h_{01}$, $h_{02}$, $h_{03}$, $h_{12}$, $h_{13}$, $h_{23}$). We point out that this null space structure exists for the gauge equivalent form $L = -\partial_\lambda h_{\alpha\beta}\partial^\lambda h^{\alpha\beta}$ on the lattice, but we included the mixing terms for comparison with the particular gauge choice in the photon case.

# The Standard Model

Strictly speaking, when finding the gradient of a vector field on the graph as we did with the Dirac operator, we need to specify a means of parallel transport. So, in our view and that of LGT, local gauge invariance is seen as a modification to the matter field gradient on the graph required by parallel transport per $U_\mu$, i.e.,

$$\gamma^\mu D_\mu \psi = \gamma^0 \left( \frac{U_0 \tilde{\psi}_0 - \psi}{ct} \right) + \gamma^1 \left( \frac{U_1 \tilde{\psi}_1 - \psi}{x} \right) + \ldots$$

where $\tilde{\psi}_i$ is the complex vector field on the node adjacent to $\psi$ in the positive $i^{\text{th}}$

direction. The Lagrangian density $L = \frac{1}{2}\overline{\psi}\left(\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta}$ is therefore seen

as the addition of parallel transport $U_\mu$ and a curvature term $A^\dagger\left(\partial_2{}^\dagger \partial_2\right)A$, where $A$

generates $U_\mu$, to $L = \frac{1}{2}\overline{\psi}\left(\sqrt{\partial_1{}^\dagger \partial_1}\right)\psi$ to produce a well-defined field gradient between $\tilde{\psi}_i$

and $\psi$. Thus, the action of the Standard Model results from the self-consistent co-construction of space, time and sources via field gradients on the graph as ultimately underwritten by the SCC.

If one introduces two vectors at each node, this same standard requires

$$\gamma^{\mu} D_{\mu}\psi = \gamma^{0}\left(\frac{\begin{bmatrix} C_{011} & C_{012} \\ C_{021} & C_{022} \end{bmatrix}\begin{bmatrix} \tilde{\psi}_0^1 \\ \tilde{\psi}_0^2 \end{bmatrix} - \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}}{ct}\right) + \gamma^{1}\left(\frac{\begin{bmatrix} C_{111} & C_{112} \\ C_{121} & C_{122} \end{bmatrix}\begin{bmatrix} \tilde{\psi}_1^1 \\ \tilde{\psi}_1^2 \end{bmatrix} - \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}}{x}\right) + \ldots$$

where the matrix $C_{\mu ab}$ is an element of SU(2) associated with the link in the positive $\mu^{th}$ direction from $\begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}$. Again, we have the same form for our field gradients, i.e., the nodal field gradients weighted by the link field, which still contributes a gradient to the action $-\frac{1}{4g^2}\left(F^{a\alpha\beta} F^{a}_{\alpha\beta}\right)$ (sum over $a$) where $g$ is the coupling constant, $F^{a}_{\alpha\beta} = \partial_{\alpha} A^{a}_{\beta} - \partial_{\beta} A^{a}_{\alpha} + f^{abc} A^{b}_{\alpha} A^{c}_{\beta}$ (sum over $b$ and $c$) and $f^{abc}$ are the structure constants of SU(2). The pattern is extended to SU(3) for three vectors at each node and all possible mixing between U(1), SU(2) and SU(3) forms the Standard Model.

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$
$$\frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$
$$M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H -$$
$$\frac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2} M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} +\right.$$
$$\left.\frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$
$$\frac{1}{8} g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$$
$$gMW^+_\mu W^-_\mu H - \frac{1}{2} g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2} ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2} g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$
$$\phi^+\partial_\mu H)] + \frac{1}{2} g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$
$$igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4} g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$
$$\frac{1}{4} g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac{1}{2} g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) - \frac{1}{2} ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2} g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \frac{1}{2} ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial +$$
$$m_d^\lambda)d^\lambda_j + igs_wA_\mu\left[-(\bar{e}^\lambda\gamma e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma d^\lambda_j)\right] + \frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 +$$
$$\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3} s_w^2 - 1 - \gamma^5)u^\lambda_j) +$$
$$(\bar{d}^\lambda_j\gamma^\mu(1-\tfrac{8}{3} s_w^2-\gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1 +$$
$$\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)] +$$
$$\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) +$$
$$i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1 +$$
$$\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] -$$
$$\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) +$$
$$\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2-M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y +$$
$$igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) +$$
$$igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) +$$
$$igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) -$$
$$\frac{1}{2} gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ -$$
$$\bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ -$$
$$\bar{X}^0X^+\phi^-] + \frac{1}{2} igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]$$

Lagrangian density for Standard Model
Credit: T.D. Gutierrez

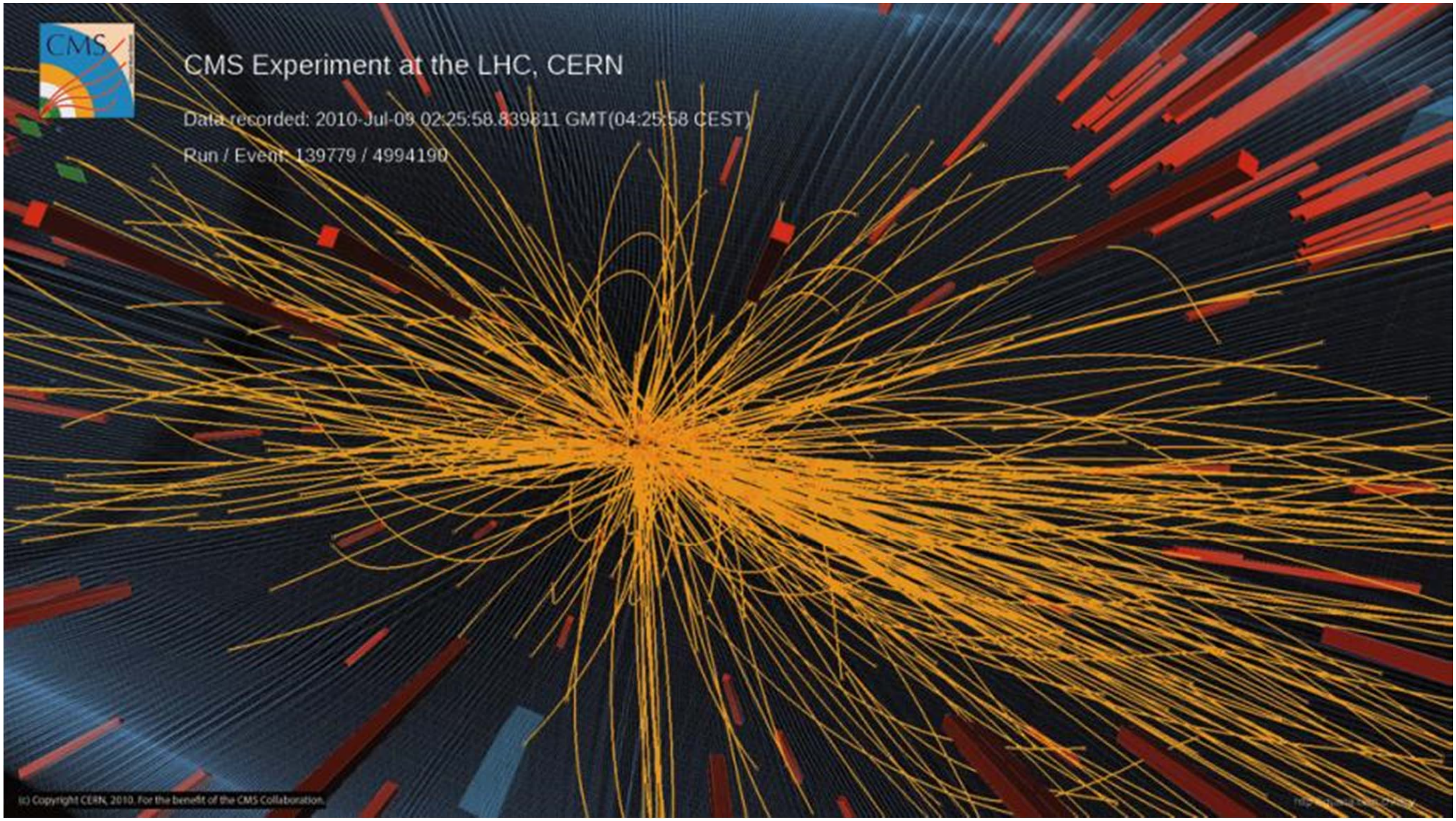
With this understanding of the Standard Model, we see that the next logical addition to our collection of fundamental spacetimesource elements would be those constructed from the gradient of vector fields on links. The scalar field on plaquettes (basis for quantum gravity) would define parallel transport for this field gradient in the manner scalar fields on links defines parallel transport for the vector fields on nodes. Thus, underwriting TTOs via spacetimesource elements leads to a relatively simple picture of unification (next slide) compared to that based on fundamental particles (previous slide). However, while we do not view particle physics as the study of what is ultimately fundamental in Nature, it has been essential to understanding how the fundamental elements of spacetimesource are to be combined, and what properties are represented by $\bar{J}$.

# Unification and Quantum Gravity per Theory X: The fundamental elements of spacetimesource

| | | |
|---|---|---|
| Scalar field on nodes | One vector each node | One vector each link |
| Scalar field on links | Two vectors each node | Two vectors each link |
| Scalar field on plaquettes | Three vectors each node | Three vectors each link |

The major questions to be answered in this view of unification are clear. Is there a limit to the number of vectors that can be (or need be) introduced on nodes and links? If so, does it have to do with information density? Is it related to quark confinement? Or, is there a purely mathematical fact that underwrites it? Why is there no physical counterpart to a scalar field on cubes? Is this because it requires (4+1)D to close graphically and satisfy the boundary of a boundary principle for all graphical entities? What physical objects correspond to vector fields on links? Are they just quarks and leptons interacting gravitationally? Or, will this generate new fermions that only interact gravitationally, e.g., dark matter? How many terms in the lattice Einstein-Hilbert action are truly needed to account for all observed phenomena, i.e., how much of GR will remain? Will we need sources that are functions of $h_{\alpha\beta}$? Obviously, the program of unification changes non-trivially in this approach. We next explain particle physics per theory X.

In our approach, the role of the field is very different than in QFT where it pervades otherwise empty, continuous space to mediate the exchange of matter-energy between sources. Per theory X (and that of LGT), a field is simply a map of scalars and vectors to the graph. One obtains QFT results from LGT by letting the lattice spacing go to zero. In fact, one can understand QFT renormalization through this process of lattice regularization(33). As it turns out, however, this limit does not always exist, so calculated values are necessarily obtained from small, but non-zero, lattice spacing. With this picture in mind, we can say simply what we are proposing: The lattice is fundamental, not its continuum limit. Once one accepts this premise, it's merely a matter of degree to have large spacetimesource elements, which is the basis for our explanation of the twin-slit experiment and dark energy (below). In this approach, *there is no graphical counterpart to "quantum systems" traveling through space as a function of time from Source to sink to "cause" detector clicks.* This implies the empirical goal at the fundamental level is to tell a unified story about detector events to include individual clicks – how they are distributed in space (e.g., interference patterns, interferometer outcomes, spin measurements), how they are distributed in time (e.g., click rates, coincidence counts), how they are distributed in space and time (e.g., particle trajectories), and how they generate more complex phenomena (e.g., photoelectric effect, superconductivity). Thus in theory X, particle physics per QFT is in the business of characterizing large sets of detector data, i.e., all the individual clicks.


CMS
CMS Experiment at the LHC, CERN
Data recorded: 2010-Jul-09 02:25:58.839811 GMT(04:25:58 CEST)
Run / Event: 139779 / 4994190
(c) Copyright CERN, 2010. For the benefit of the CMS Collaboration.

Implications for Quantum Physics?
Particle Physics not fundamental

RBW → Modification to General Relativity
Modify GR via Regge calculus

Start with easiest GR solution, Einstein-de Sitter cosmology (EdS)
Photon exchange over cosmological distances, type Ia supernova

Union2 Compilation data is distance modulus vs redshift to $z = 1.4$

$$\mu = 5\log\left(\frac{D_L}{10pc}\right)$$

Linear regression $\log\left(\frac{D_L}{Gpc}\right)$ vs log(z) SSE = 1.95 and R = .9955

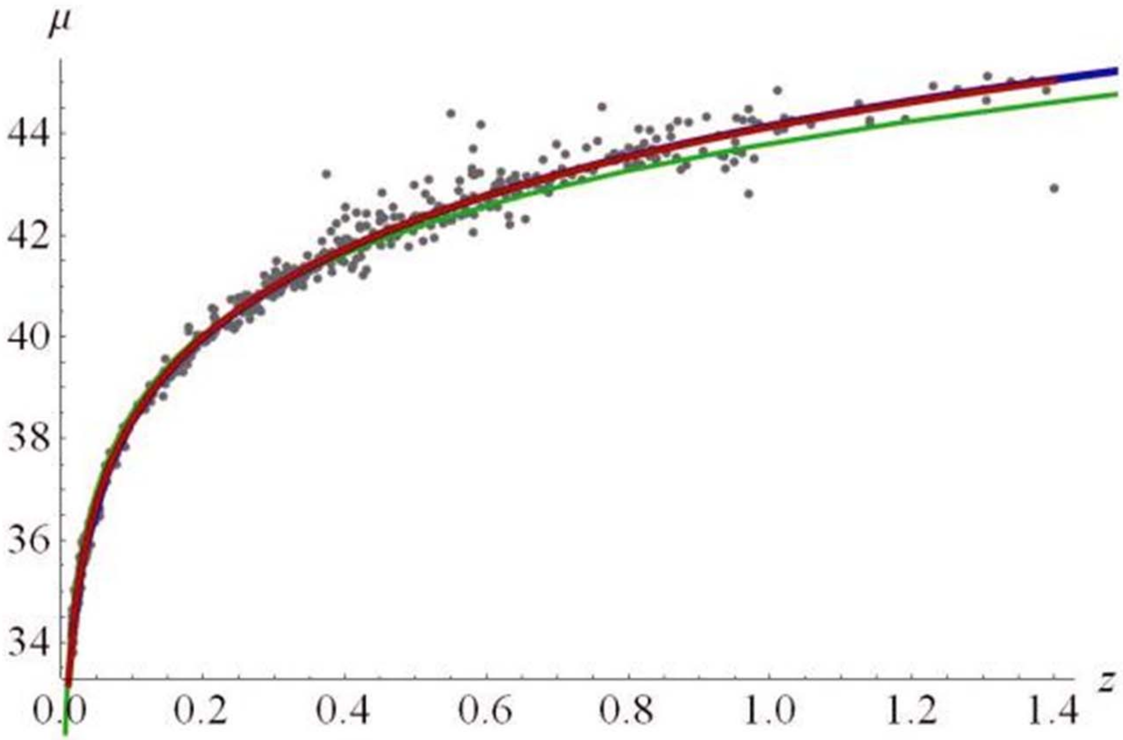
$$D_L = (1+z)d_p$$

$$d_p = c\int_{t_e}^{t_o}\frac{dt}{a(t)} = c\int_{a_e}^{1}\frac{da}{\dot{a}(t)} = 3ct_o\left(1-\frac{1}{\sqrt{1+z}}\right)$$

Best fit EdS SSE = 2.68 using $H_o$ = 60.9 km/s/Mpc ($t_o$ = 10.7 Gy)

Current (2011) "best estimate" $H_o$ = (73.8 ± 2.4) km/s/Mpc

Best fit ΛCDM SSE = 1.79 using $\Omega_\Lambda = 0.71$, $\Omega_M = 0.29$ and
$H_o$ = 69.2 km/s/Mpc




Plot of Union2 Compilation data of distance moduli $\mu$ versus redshifts $z$ for type Ia supernovae. Superimposed are the best fits for EdS (green), ΛCDM (blue), and MORC (red). The MORC curve is terminated at $z$ = 1.4 in this figure so that the ΛCDM curve is visible underneath.

# Regge Calculus

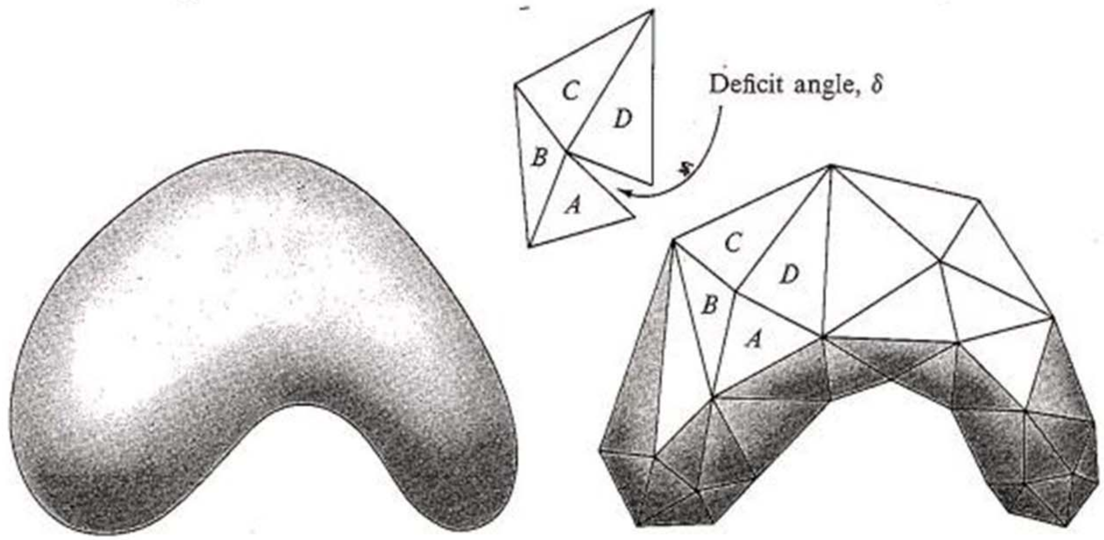



**Figure 42.1.**
A 2-geometry with continuously varying curvature can be approximated arbitrarily closely by a polyhedron built of triangles, provided only that the number of triangles is made sufficiently great and the size of each sufficiently small. The geometry in each triangle is Euclidean. The curvature of the surface shows up in the amount of deficit angle at each vertex (portion *ABCD* of polyhedron laid out above on a flat surface).

Reproduced from Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. W.H. Freeman, San Francisco (1973), p. 1168.

Hilbert action for a 4D vacuum lattice

$$I_R = \frac{1}{8\pi} \sum_{\sigma_j \in L} \varepsilon_i A_i$$

The counterpart to Einstein's equations

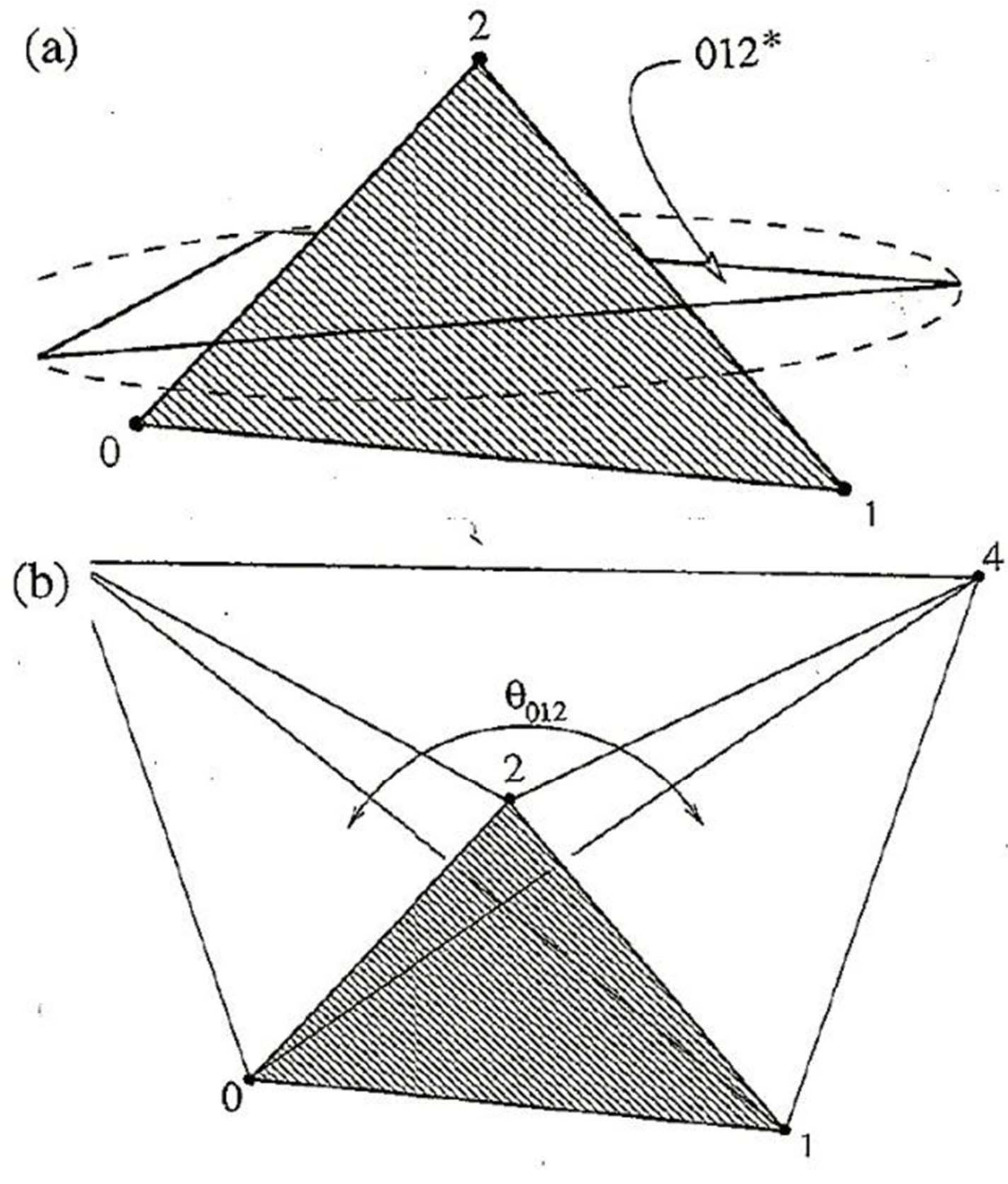
$$\frac{\delta I_R}{\delta \ell_j^2} = 0$$

$$\frac{\delta I_R}{\delta \ell_j^2} = -\frac{\delta I_{M-E}}{\delta \ell_j^2}$$

Stress-energy tensor is associated lattice edges

Regge's equations are to be satisfied for any particular choice of the two tensors on the lattice.

Thus, Regge's equations are, like Einstein's equations, a self-consistency criterion for the stress-energy tensor and metric.


(a)
2
012*
0
1
(b)
4
θ012
2
0
1

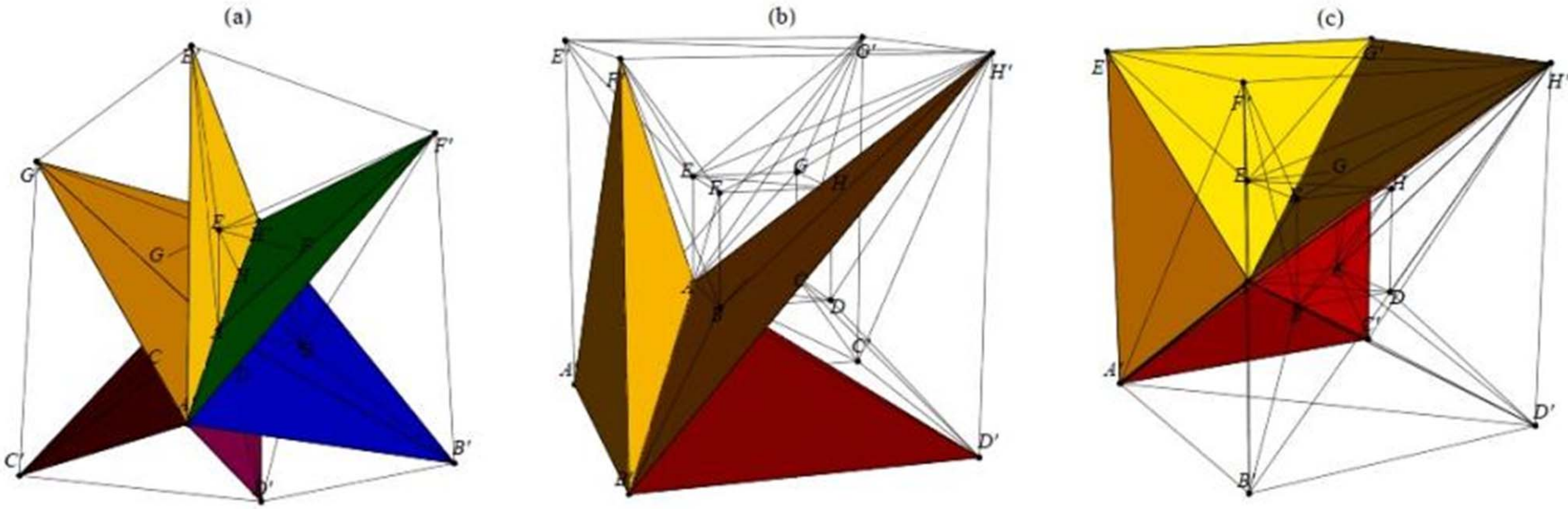



**Figure 5.** (a) Triangle $AA'H'$ and its entourage. (b) Triangle $AA'B'$ and part of its entourage: $AA'B'D'$, $AA'B'H'$, $AA'B'F'$. (c) Triangle $AA'G'$ and part of its entourage: $AA'G'C'$, $AA'G'H'$, $AA'G'E'$

Following Brewin[51] and Gentle[52], we take the stress energy in the $AA'$ edge to be of the form

$$\frac{12Gm}{c^2(ic\Delta t)}$$

Given there are six triangles of the type $AA'B'$ for the edge $AA'$, our Regge equation is

$$\frac{12iR(a_n+a_{n+1})}{c\Delta t}\frac{\left(\pi-\cos^{-1}\left(\frac{\left(\frac{R}{c}\right)^2\left(\frac{a_{n+1}-a_n}{\Delta t}\right)^2}{2\left(\left(\frac{R}{c}\right)^2\left(\frac{a_{n+1}-a_n}{\Delta t}\right)^2+2\right)}\right)-2\cos^{-1}\left(\frac{\sqrt{3\left(\frac{R}{c}\right)^2\left(\frac{a_{n+1}-a_n}{\Delta t}\right)^2+4}}{2\sqrt{\left(\frac{R}{c}\right)^2\left(\frac{a_{n+1}-a_n}{\Delta t}\right)^2+2}}\right)\right)}{\sqrt{\left(\frac{R}{c}\right)^2\left(\frac{a_{n+1}-a_n}{\Delta t}\right)^2+4}}$$

$$=\frac{12iGm}{c^3\Delta t}$$

$$\frac{\pi - \cos^{-1}\left(\frac{v^2/c^2}{2(v^2/c^2+2)}\right) - 2\cos^{-1}\left(\frac{\sqrt{3v^2/c^2+4}}{2\sqrt{v^2/c^2+2}}\right)}{\sqrt{v^2/c^2+4}} = \frac{Gm}{2rc^2}$$

which we emphasize is unmodified Regge calculus. If $v^2/c^2 \ll 1$, then a power series expansion of the LHS gives

$$\frac{v^2}{4c^2} + \mathcal{O}\left(\frac{v}{c}\right)^4 = \frac{Gm}{2rc^2}$$

Thus, to leading order, our Regge EdS is EdS, i.e., $\frac{v^2}{2} = \frac{Gm}{r}$, which is just a Newtonian conservation of energy expression for an unit mass moving at escape velocity $v$ at distance $r$ from mass $m$.

$$D_L = (1+z)\sqrt{\vec{D}_p \cdot \vec{D}_p}$$

$$ds^2 = -c^2 dt^2 + dD_p^2$$

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

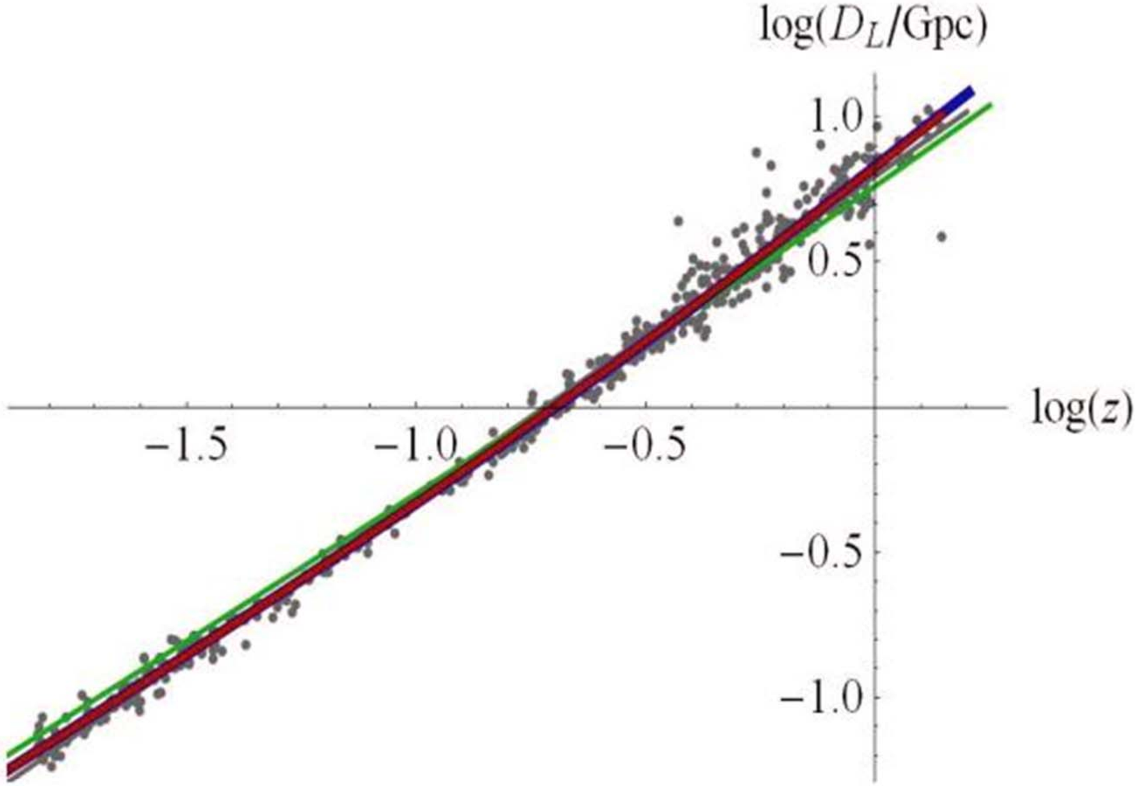
$$\vec{D}_p \cdot \vec{D}_p = (1 + h_{11}) D_p^2$$

$$\partial^2 h_{\mu\nu} \propto \left( T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T_\alpha^\alpha \right)$$

$$h_{11} = \mathrm{A} D_p + \mathrm{B}$$

Best fit MORC SSE = 1.77 using R = $A^{-1}$ = 8.38 Gcy and

$m$ = 1.71 x $10^{52}$ kg. This gives $H_o$ = 73.9 km/s/Mpc




Plot of transformed Union2 data along with the best fits for linear regression (gray), EdS (green), ΛCDM (blue), and MORC (red).

Best fit line through $\log(D_L/\text{Gpc})$ versus $\log(z)$ gives a correlation of 0.9955 and a sum of squares error (SSE) of 1.95.

The best fit ΛCDM gives SSE = 1.79 using Ho = 69.2 km/s/Mpc, $\Omega_M$ = 0.29 and $\Omega_\Lambda$ = 0.71.

The parameters for ΛCDM yielding the most robust fit to[1] "the Wilkinson Microwave Anisotropy Probe data with the latest distance measurements from the Baryon Acoustic Oscillations in the distribution of galaxies and the Hubble constant measurement" are Ho = 70.3 km/s/Mpc, $\Omega_M$ = 0.27 and $\Omega_\Lambda$ = 0.73.

The best fit MORC gives SSE = 1.77 and Ho = 73.9 km/s/Mpc using $A^{-1}$ = 8.38 Gcy and $m = 1.71 \times 10^{52}$ kg.

The best fit EdS gives SSE = 2.68 using Ho = 60.9 km/s/Mpc.

[1] Komatsu, E., et al.: Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. http://arxiv.org/abs/1001.4538. (2010)

Can dark matter be eliminated in this fashion?