

# A Principle Explanation of Bell State Entanglement: Conservation per No Preferred Reference Frame

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## Abstract

Many in quantum foundations seek a principle explanation of Bell spin state (“Bell state” for short) entanglement. While reconstructions of quantum mechanics (QM) have been produced, the community does not find them compelling. Herein we offer a principle explanation for Bell state entanglement, i.e., conservation per no preferred reference frame (NPRF), such that NPRF unifies Bell state entanglement with length contraction and time dilation from special relativity (SR). What makes this a principle explanation is that it’s grounded directly in phenomenology, it is an adynamical and acausal explanation that involves adynamical global constraints as opposed to dynamical laws or causal mechanisms, and it’s unifying with respect to QM and SR.

## 1 Introduction

Many physicists in quantum information theory (QIT) are calling for “clear physical principles” [Fuchs and Stacey, 2016] to account for quantum mechanics (QM). As [Hardy, 2016] points out, “The standard axioms of [quantum theory] are rather ad hoc. Where does this structure come from?” Fuchs points to the postulates of special relativity (SR) as an example of what QIT seeks for QM [Fuchs and Stacey, 2016] and SR is a principle theory [Felline, 2011]. That is, the postulates of SR are constraints offered without a corresponding constructive explanation. In what follows, [Einstein, 1919] explains the difference between the two:

We can distinguish various kinds of theories in physics. Most of them are constructive. They attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple

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formal scheme from which they start out. ...

Along with this most important class of theories there exists a second, which I will call “principle-theories.” These employ the analytic, not the synthetic, method. The elements which form their basis and starting point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy. ...

The advantages of the constructive theory are completeness, adaptability, and clearness, those of the principle theory are logical perfection and security of the foundations. The theory of relativity belongs to the latter class.

It is worth noting the irony that in the past two decades, just as some have sought a principle explanation of QM, others have sought a constructive explanation of SR [Brown, 2005, Brown and Pooley, 2006]. While we cannot go into details on such matters, we note that reasons for seeking a principle explanation of QM include not just the ad hoc nature of the postulates, but the fact that there is no agreement on “constructive interpretations,” in part because they do nothing but recover what is already in textbook QM, and therefore lead to no new physics or unification. Indeed, non-local interpretations of QM only make unification with SR more problematic.

For those who believe the fundamental explanation for QM phenomena must be constructive, at least in the sense envisioned by Einstein above, none of the mainstream interpretations neatly fit the bill. Not only do most interpretations entail some form of QM holism, contextuality, and/or non-locality, the remainder invoke priority monism and/or multiple branches or outcomes. The problem with attempting a constructive account of QM is, as articulated by [Van Camp, 2011], “Constructive interpretations are attempted, but they are not unequivocally constructive in any traditional sense.” Thus, [Van Camp, 2011] states:

The interpretive work that must be done is less in coming up with a constructive theory and thereby explaining puzzling quantum phenomena, but more in explaining why the interpretation counts as explanatory at all given that it must sacrifice some key aspect of the traditional understanding of causal-mechanical explanation.

It seems clear all of this would be anathema to Einstein and odious with respect to constructive explanation, especially if say, statistical mechanics is the paradigm example of constructive explanation. Thus, for many it seems wise to at least attempt a principle explanation of QM, as sought by QIT. The problem with QIT’s attempts is noted by [Van Camp, 2011]:

<b>Why the quantum? = Why the Tsirelson bound?</b>		
	CHSH Quantity	
-2 ↔ 2	$-2\sqrt{2} \leftrightarrow 2\sqrt{2}$	PR correlations → 4
Satisfy Bell inequality	Tsirelson bound	No-signaling max
Classical Correlations	Quantum Correlations	Superquantum Correlations
Violate Constraint	Satisfy Constraint	Violate Constraint

Figure 1: **Answer to Bub’s question.** The “constraint” is conservation per no preferred reference frame.

However, nothing additional has been shown to be incorporated into an information-theoretic reformulation of quantum mechanics beyond what is contained in quantum mechanics itself. It is hard to see how it could offer more unification of the phenomena than quantum mechanics already does since they are equivalent, and so it is not offering any explanatory value on this front.

Nonetheless, QIT continues to seek “the *reconstruction* of quantum theory” via a constraint-based/principle approach [Chiribella and Spekkens, 2016]. Indeed, QIT has produced several different sets of axioms, postulates, and “physical requirements” in terms of quantum information, which all reproduce quantum theory. Along those lines, [Bub, 2004, Bub, 2012, Bub, 2016] has asked, “why is the world quantum and not classical, and why is it quantum rather than superquantum, i.e., why the Tsirelson bound for quantum correlations?”

Despite all the success of QIT, the community does not find any of the reconstructions compelling. [Cuffaro, 2017], for example, argues that information causality needs to be justified in some physical sense. And, as Hardy states, “When I started on this, what I wanted to see was two or so obvious, compelling axioms that would give you quantum theory and which no one would argue with” [Ball, 2017]. Fuchs quotes Wheeler, “If one really understood the central point and its necessity in the construction of the world, one ought to state it in one clear, simple sentence” [Fuchs and Stacey, 2016, p. 302]. Asked if he had such a sentence, Fuchs responded, “No, that’s my big failure at this point” [Fuchs and Stacey, 2016, p. 302]. As we will show, the same principle responsible for the postulates of SR is also responsible for Bell state entanglement which uniquely produces the Tsirelson bound [Cirel’son, 1980, Landau, 1987, Khalfin and Tsirelson, 1992], viz., no preferred reference frame (NPRF) (Figure 1).

The term “reference frame” has many meanings in physics related to microscopic and macroscopic phenomena, Galilean versus Lorentz transformations, relatively moving observers, etc. The difference between Galilean and Lorentz

transformations resides in the fact that the speed of light is finite, so NPRF entails the light postulate of special relativity, i.e., that everyone measure the same speed of light  $c$ , regardless of their motion relative to the source. If there was only one reference frame for a source in which the speed of light equaled the prediction from Maxwell’s equations ( $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ ), then that would certainly constitute a preferred reference frame. Herein, we extend NPRF to include the measurement of another fundamental constant of nature, Planck’s constant  $h$  ( $= 2\pi\hbar$ ).

As Steven Weinberg points out, measuring an electron’s spin via Stern-Gerlach (SG) magnets constitutes the measurement of “a universal constant of nature, Planck’s constant” [Weinberg, 2017, p. 3] (Figure 2). So if NPRF applies equally here, everyone must measure the same value for Planck’s constant  $h$  regardless of their SG magnet orientations relative to the source, which like the light postulate is an empirical fact. By “relative to the source” of a pair of spin-entangled particles, we mean relative “to the vertical in the plane perpendicular to the line of flight of the particles” [Mermin, 1981, p. 943] (Figure 3). Here the possible spin outcomes  $\pm \frac{\hbar}{2}$  represent a fundamental (indivisible) unit of information per Dakic and Brukner’s first axiom in their reconstruction of quantum theory, “An elementary system has the information carrying capacity of at most one bit” [Dakic and Brukner, 2009]. Thus, different SG magnet orientations relative to the source constitute different “reference frames” in quantum mechanics just as different velocities relative to the source constitute different “reference frames” in special relativity. Borrowing from [Einstein, 1936], NPRF might be stated:

No one’s “sense experiences,” to include measurement outcomes, can provide a privileged perspective on the “real external world.”

This is consistent with the notion of symmetries per [Hicks, 2019]:

There are not two worlds in one of which I am here and in the other I am three feet to the left, with everything else similarly shifted. Instead, there is just this world and two mathematical descriptions of it. The fact that those descriptions put the origin at different places does not indicate any difference between the worlds, as the origin in our mathematical description did not correspond to anything in the world anyway. The symmetries tell us what structure the world does not have.

That is, there is just one “real external world” harboring many, but always equal perspectives as far as the physics is concerned.

We have shown elsewhere that the quantum correlations and quantum states corresponding to the Bell states, which uniquely produce the Tsirelson bound for the Clauser–Horne–Shimony–Holt (CHSH) quantity, can be derived from conservation per NPRF [Stuckey et al., 2019]. Thus, Bell state entanglement is ultimately grounded in NPRF, just as SR [Stuckey et al., 2020]. As summarized in Figure 1, the quantum correlations responsible for the Tsirelson bound satisfy

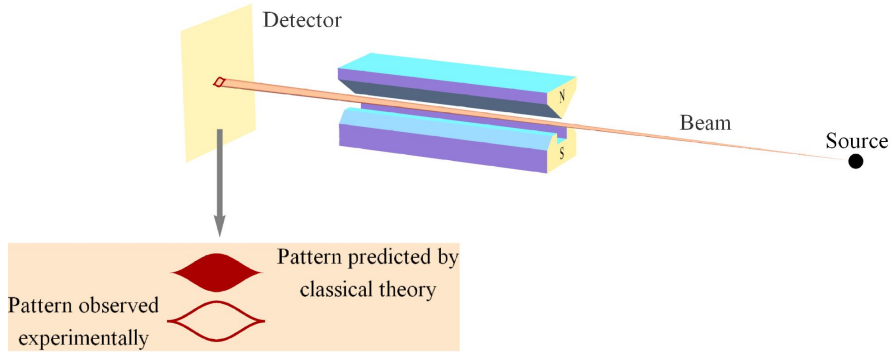


Figure 2: A Stern-Gerlach (SG) spin measurement showing the two possible outcomes, up ( $+\frac{\hbar}{2}$ ) and down ( $-\frac{\hbar}{2}$ ) or  $+1$  and  $-1$ , for short. The important point to note here is that the classical analysis predicts all possible deflections, not just the two that are observed. This binary (quantum) outcome reflects Dakic and Brukner’s first axiom in their reconstruction of quantum theory, “An elementary system has the information carrying capacity of at most one bit” [Dakic and Brukner, 2009]. The difference between the classical prediction and the quantum reality uniquely distinguishes the quantum joint distribution from the classical joint distribution for the Bell spin states [Garg and Mermin, 1982].

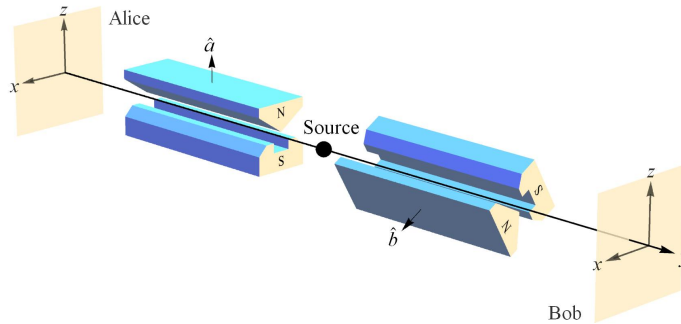


Figure 3: Alice and Bob making spin measurements on a pair of spin-entangled particles with their Stern-Gerlach (SG) magnets and detectors in the  $xz$ -plane. Here Alice and Bob’s SG magnets are not aligned so these measurements represent different reference frames. Since their outcomes satisfy Dakic and Brukner’s Axiom 1 in all reference frames and satisfy explicit conservation of spin angular momentum in the same reference frame, they can only satisfy conservation of spin angular momentum on *average* in different reference frames.

conservation per NPRF while both classical and superquantum correlations can violate this constraint. Therefore a principle explanation of Bell state entanglement and the Tsirelson bound that be stated in “one clear, simple sentence” is “conservation per no preferred reference frame” (Figure 1).

What qualifies as a principle explanation versus constructive turns out to be a fraught and nuanced question [Felline, 2011] and we do not want to be sidetracked on that issue as such. Let us therefore state explicitly that what makes our explanation a principle one is that it is grounded directly in phenomenology, it is an adynamical and acausal explanation that involves adynamical global constraints as opposed to dynamical laws or causal mechanisms, and it is unifying with respect to QM and SR.

Let us also note that while contrary to certain others [Brown, 2005, Brown and Pooley, 2006, Norton, 2008, Menon, 2019], we are arguing that conservation per NPRF need not ever be discharged by a constructive explanation or interpretation. This is at least partially distinct from the question in SR for example, of whether facts about physical geometry are grounded in facts about dynamical fields or vice-versa. Furthermore, this principle explanation is consistent with any number of “constructive interpretations” of QM. For example, this principle explanation avoids the complaints about Bub’s proposed principle explanation of QM leveled by [Felline, 2018]. That is, the principle being posited herein does not require a solution to the measurement problem nor again does it necessarily beg for a constructive counterpart.

In Section 2 we provide a quick review of length contraction, time dilation, the relativity of simultaneity, and Lorentz transformations per SR. In Section 3 we explain how conservation per NPRF is responsible for Bell state entanglement in complete analogy with the light postulate. In Section 4 we argue that principle explanation for these mysteries suffices despite the fact that there is no constructive counterpart.

## 2 NPRF and Special Relativity

Suppose there are three women moving together at  $0.6c$  with respect to two men. The men and women agree on the details of the following four Events (men’s coordinates are lower case and women’s coordinates are upper case):

- Event 1: Joe meets Sara at  $X_1 = x_1 = 0, T_1 = t_1 = 0$ .
- Event 2: Bob meets Kim at  $X_2 = 1250\text{km}, T_2 = -0.0025\text{s}, x_2 = 1000\text{km}, t_2 = 0$ .
- Event 3: Bob meets Alice at  $X_3 = 800\text{km}, T_3 = 0, x_3 = 1000\text{km}, t_3 = 0.002\text{s}$ .
- Event 4: Bob meets Sara at  $X_4 = 0, T_4 = 0.0044\text{s}, x_4 = 1000\text{km}, t_4 = 0.0055\text{s}$ .

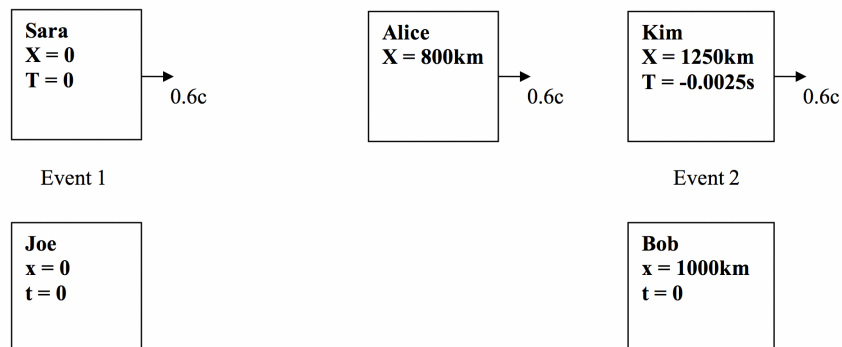


Figure 4: Events 1 and 2 are simultaneous for the men and are spaced at a distance of 1000km. The women say the distance between Sara and Kim is 1250km. Thus, the men say the women’s meter sticks are short.

The lower-case and upper-case coordinates for each Event are related by Lorentz transformations with  $\gamma = 1.25$ . Here is the story according to the men.

The women are moving in the positive  $x$  direction at  $0.6c$ . Events 1 and 2 are simultaneous ( $t_1 = t_2 = 0$ ), so the distance between Sara and Kim is  $x_2 = 1000\text{km}$ . The women say the distance between Sara and Kim is  $X_2 = 1250\text{km}$ , so their proper distance has been length contracted by  $\gamma$  (Figure 4). Event 4 happens  $t_4 = 0.0055\text{s}$  after Events 1 and 2, but Sara’s clock has only ticked off  $T_4 = 0.0044\text{s}$ , so her proper time has been dilated by a factor of  $\gamma$  (Figure 5). Therefore, the men say the women’s meter sticks are short (length contraction) and the women’s clocks are running slow (time dilation). Here is the story according to the women.

The men are moving in the negative  $X$  direction at  $0.6c$ . Events 1 and 3 are simultaneous ( $T_1 = T_3 = 0$ ), not Events 1 and 2 as the men claim (relativity of simultaneity). Thus, the distance between Joe and Bob is  $X_3 = 800\text{km}$ , not  $x_3 = 1000\text{km}$  as the men claim (Figure 6). Again, the proper distance has been length contracted by  $\gamma$ . Event 3 happens  $0.0025\text{s}$  after Event 2, but Bob’s clock has only ticked off  $t_3 = 0.002\text{s}$ , so his proper time has been dilated by a factor of  $\gamma$  (Figure 6). Therefore, the women say the men’s meter sticks are short and the men’s clocks are running slow.

In summary, NPRF gives the postulates of SR whence the Lorentz transformations, time dilation, length contraction, and the relativity of simultaneity. Since Alice and Bob always measure the same speed of light  $c$  regardless of their relative motion per NPRF, Alice says Bob’s temporal and spatial measurements need to be corrected per time dilation and length contraction while Bob says the same thing about Alice’s measurements. But, if NPRF is true and fundamental, then neither need to be corrected (relativity of simultaneity). Thus, the mysteries of length contraction and time dilation in SR ultimately reside



Figure 5: According to the men, Event 4 happens  $t_4 = 0.0055\bar{s}$  after Events 1 and 2, but Sara's clock has only ticked off  $T_4 = 0.0044s$ . Thus, the men say the women's clocks are running slow.

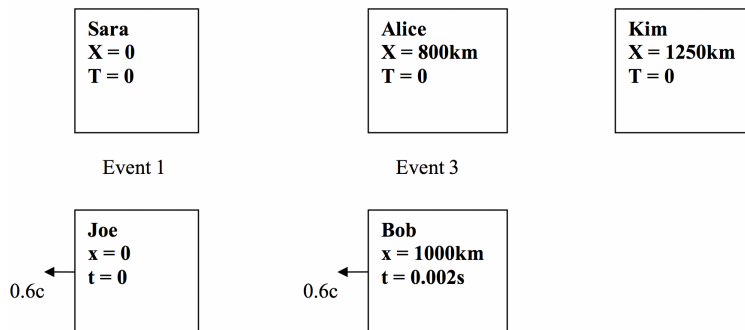


Figure 6: According to the women, Events 1 and 3 are simultaneous not Events 1 and 2 as the men claim (relativity of simultaneity). Thus, the distance between Joe and Bob is  $X_3 = 800\text{km}$ , not  $x_3 = 1000\text{km}$  as the men claim, i.e., the women say the men's meter sticks are short. Also, Event 3 happens  $0.0025s$  after Event 2, but Bob's clock has only ticked off  $t_3 = 0.002s$ , so the women say the men's clocks are running slow.



in NPRF starting with the fact that everyone measures the same value for the fundamental constant  $c$ . Now let us relate this mystery to the mystery of Bell state entanglement in QM.

### 3 Conservation per NPRF in QM

The Bell states are

$$\begin{aligned}
|\psi_{-}\rangle &= \frac{|ud\rangle - |du\rangle}{\sqrt{2}} \\
|\psi_{+}\rangle &= \frac{|ud\rangle + |du\rangle}{\sqrt{2}} \\
|\phi_{-}\rangle &= \frac{|uu\rangle - |dd\rangle}{\sqrt{2}} \\
|\phi_{+}\rangle &= \frac{|uu\rangle + |dd\rangle}{\sqrt{2}}
\end{aligned} \tag{1}$$

in the eigenbasis of  $\sigma_z$ . The first state  $|\psi_{-}\rangle$  is called the “spin singlet state” and it represents a total conserved spin angular momentum of zero ( $S = 0$ ) for the two particles involved. The other three states are called the “spin triplet states” and they each represent a total conserved spin angular momentum of one ( $S = 1$ , in units of  $\hbar = 1$  for spin- $\frac{1}{2}$  particles). In all four cases, the entanglement represents the conservation of spin angular momentum for the process creating the state.

If Alice is making her spin measurement  $\sigma_1$  in the  $\hat{a}$  direction and Bob is making his spin measurement  $\sigma_2$  in the  $\hat{b}$  direction, we have

$$\begin{aligned}
\sigma_1 &= \hat{a} \cdot \vec{\sigma} = a_x \sigma_x + a_y \sigma_y + a_z \sigma_z \\
\sigma_2 &= \hat{b} \cdot \vec{\sigma} = b_x \sigma_x + b_y \sigma_y + b_z \sigma_z
\end{aligned} \tag{2}$$

The correlation functions are given by [Stuckey et al., 2020]

$$\begin{aligned}
\langle \psi_{-} | \sigma_1 \sigma_2 | \psi_{-} \rangle &= -a_x b_x - a_y b_y - a_z b_z \\
\langle \psi_{+} | \sigma_1 \sigma_2 | \psi_{+} \rangle &= a_x b_x + a_y b_y - a_z b_z \\
\langle \phi_{-} | \sigma_1 \sigma_2 | \phi_{-} \rangle &= -a_x b_x + a_y b_y + a_z b_z \\
\langle \phi_{+} | \sigma_1 \sigma_2 | \phi_{+} \rangle &= a_x b_x - a_y b_y + a_z b_z
\end{aligned} \tag{3}$$

The spin singlet state is invariant under all three SU(2) transformations meaning we obtain opposite outcomes ( $\frac{1}{2} ud$  and  $\frac{1}{2} du$ ) for SG magnets at any  $\hat{a} = \hat{b}$  (Figures 2 & 3) and a correlation function of  $-\cos(\theta)$  in any plane of physical space, where  $\theta$  is the angle between  $\hat{a}$  and  $\hat{b}$  (Eq. (3)). We see that the conserved spin angular momentum ( $S = 0$ ), being directionless, is conserved in

any plane of physical space. Again,  $\hat{a} = \hat{b}$  means Alice and Bob are in the same reference frame.

The invariance of each of the spin triplet states under its respective SU(2) transformation in Hilbert space represents the SO(3) invariant conservation of spin angular momentum  $S = 1$  for each of the planes  $xz$  ( $|\phi_+\rangle$ ),  $yz$  ( $|\phi_-\rangle$ ), and  $xy$  ( $|\psi_+\rangle$ ) in physical space. Specifically, when the SG magnets are aligned (the measurements are being made in the same reference frame) anywhere in the respective plane of symmetry the outcomes are always the same ( $\frac{1}{2} uu$  and  $\frac{1}{2} dd$ ). It is a planar conservation and our experiment would determine which plane. If you want to model a conserved  $S = 1$  for some other plane, you simply create a superposition, i.e., expand in the spin triplet basis. And in that plane, you're right back to the mystery of Bell state entanglement per conserved spin angular momentum via a correlation function of  $\cos(\theta)$ , as with any of the spin triplet states (Eq. (3)).

We will explain the spin singlet state correlation function, since the spin triplet state correlation function is analogous. That we have opposite outcomes when Alice and Bob are in the same reference frame is not difficult to understand via conservation of spin angular momentum, because Alice and Bob's measured values of spin angular momentum cancel directly when  $\hat{a} = \hat{b}$  (Figure 3). But, when Bob's SG magnets are rotated by  $\theta$  relative to Alice's SG magnets, we need to clarify the situation.

We have two subsets of data, Alice's set (with SG magnets at angle  $\alpha$ ) and Bob's set (with SG magnets at angle  $\beta$ ). They were collected in  $N$  pairs (data events) with Bob's(Alice's) SG magnets at  $\alpha - \beta = \theta$  relative to Alice's(Bob's). We want to compute the correlation function for these  $N$  data events which is

$$\langle \alpha, \beta \rangle = \frac{(+1)_A(-1)_B + (+1)_A(+1)_B + (-1)_A(-1)_B + \dots}{N} \quad (4)$$

Now partition the numerator into two equal subsets per Alice's equivalence relation, i.e., Alice's +1 results and Alice's -1 results

$$\langle \alpha, \beta \rangle = \frac{(+1)_A(\sum BA+) + (-1)_A(\sum BA-)}{N} \quad (5)$$

where  $\sum BA+$  is the sum of all of Bob's results (event labels) corresponding to Alice's +1 result (event label) and  $\sum BA-$  is the sum of all of Bob's results (event labels) corresponding to Alice's -1 result (event label). Notice this is all independent of the formalism of QM. Next, rewrite Eq. (5) as

$$\langle \alpha, \beta \rangle = \frac{1}{2} (+1)_A \overline{BA+} + \frac{1}{2} (-1)_A \overline{BA-} \quad (6)$$

with the overline denoting average. Notice that to understand the quantum correlation responsible for Bell state entanglement, we need to understand the origins of  $\overline{BA+}$  and  $\overline{BA-}$  for the Bell states. We now show what that is for the spin singlet state [Unnikrishnan, 2005], the spin triplet states are analogous in their respective symmetry plane [Stuckey et al., 2020].

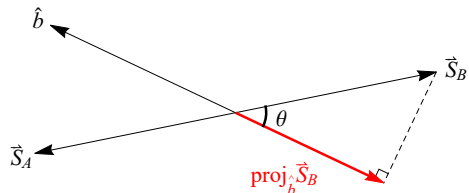


Figure 7: The spin angular momentum of Bob's particle  $\vec{S}_B = -\vec{S}_A$  projected along his measurement direction  $\hat{b}$ . This does *not* happen with spin angular momentum.

In classical physics, one would say the projection of the spin angular momentum vector of Alice's particle  $\vec{S}_A = +1\hat{a}$  along  $\hat{b}$  is  $\vec{S}_A \cdot \hat{b} = +\cos(\theta)$  where again  $\theta$  is the angle between the unit vectors  $\hat{a}$  and  $\hat{b}$ . That's because the prediction from classical physics is that all values between  $+1$  ( $\frac{\hbar}{2}$ ) and  $-1$  ( $\frac{\hbar}{2}$ ) are possible outcomes for a spin measurement (Figure 2). From Alice's perspective, had Bob measured at the same angle, i.e.,  $\beta = \alpha$ , he would have found the spin angular momentum vector of his particle was  $\vec{S}_B = -\vec{S}_A = -1\hat{a}$ , so that  $\vec{S}_A + \vec{S}_B = \vec{S}_{Total} = 0$ . Since he did not measure the spin angular momentum of his particle at the same angle, he should have obtained a fraction of the length of  $\vec{S}_B$ , i.e.,  $\vec{S}_B \cdot \hat{b} = -1\hat{a} \cdot \hat{b} = -\cos(\theta)$  (Figure 7; this also follows from counterfactual spin measurements on the single-particle state [Boughn, 2017]). Of course, Bob only ever obtains  $+1$  or  $-1$ , but suppose that Bob's outcomes *average*  $-\cos(\theta)$  (Figure 8). This means

$$\overline{BA+} = -\cos(\theta) \quad (7)$$

Likewise, for Alice's  $(-1)_A$  results we have

$$\overline{BA-} = \cos(\theta) \quad (8)$$

Putting these into Eq. (6) we obtain

$$\langle \alpha, \beta \rangle = \frac{1}{2}(+1)_A(-\cos(\theta)) + \frac{1}{2}(-1)_A(\cos(\theta)) = -\cos(\theta) \quad (9)$$

which is precisely the correlation function given by QM for the spin singlet state. Notice that Eqs. (7) & (8) are mathematical facts for obtaining the quantum correlation function, we are simply motivating these facts via conservation of spin angular momentum in accord with the SU(2) Bell state invariances. Of course, Bob could partition the data according to his equivalence relation (per his reference frame) and claim that it is Alice who must average her results (obtained in her reference frame) to conserve angular momentum (Figure 8).

We posit that the reason we have average-only conservation in different reference frames is ultimately due to NPRF. To motivate NPRF for the Bell states, consider the empirical facts. First, Bob and Alice both measure  $\pm 1$  ( $\frac{\hbar}{2}$ ) for all



Figure 8: **Average View for the Spin Singlet State.** Reading from left to right, as Bob rotates his SG magnets relative to Alice’s SG magnets for her +1 outcome, the average value of his outcome varies from  $-1$  (totally down, arrow bottom) to  $0$  to  $+1$  (totally up, arrow tip). This obtains per conservation of spin angular momentum on average in accord with no preferred reference frame. Bob can say exactly the same about Alice’s outcomes as she rotates her SG magnets relative to his SG magnets for his +1 outcome. That is, their outcomes can only satisfy conservation of spin angular momentum on average in different reference frames, because they only measure  $\pm 1$ , never a fractional result. Thus, just as with the light postulate of special relativity, we see that no preferred reference frame leads to a counterintuitive result. Here it requires quantum outcomes  $\pm 1 \left(\frac{\hbar}{2}\right)$  for all measurements and that leads to the mystery of “average-only” conservation.

SG magnet orientations, i.e. in all reference frames. In order to satisfy conservation of spin angular momentum for any given trial when Alice and Bob are making different measurements, i.e., when they are in different reference frames, it would be necessary for Bob or Alice to measure some fraction,  $\pm \cos(\theta)$ . For example, if Alice measured  $+1$  at  $\alpha = 0$  for an  $S = 1$  state (in the plane of symmetry) and Bob made his measurement (in the plane of symmetry) at  $\beta = 60^\circ$ , then Bob’s outcome would need to be  $\frac{1}{2}$  (Figure 9). In that case, we would know that Alice measured the “true” spin angular momentum of her particle while Bob only measured a component of the “true” spin angular momentum for his particle. Thus, Alice’s SG magnet orientation would definitely constitute a “preferred reference frame.”

But, this is precisely what does *not* happen. Alice and Bob both always measure  $\pm 1 \left(\frac{\hbar}{2}\right)$ , no fractions, in accord with NPRF. And, this fact alone distinguishes the quantum joint distribution from the classical joint distribution [Garg and Mermin, 1982] (Figure 2). Therefore, the average-only conservation responsible for the correlation function for the Bell states is actually conservation resulting from NPRF.

## 4 Principle versus Constructive Explanation for Bell State Entanglement

As we saw in Section 2 for SR, if Alice is moving at velocity  $\vec{V}_a$  relative to a light source, then she measures the speed of light from that source to be  $c \left(= \frac{1}{\sqrt{\mu_o \epsilon_o}}\right)$ ,

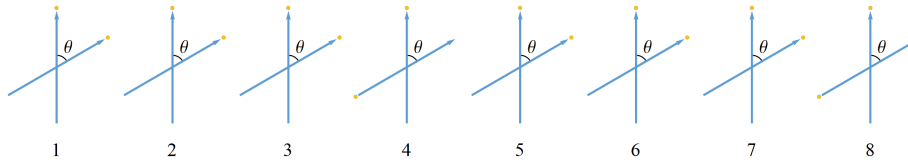


Figure 9: A spatiotemporal ensemble of 8 experimental trials for the spin triplet states showing Bob's outcomes corresponding to Alice's +1 outcomes when  $\theta = 60^\circ$ . Spin angular momentum is not conserved in any given trial, because there are two different measurements being made, i.e., outcomes are in two different reference frames, but it is conserved on average for all 8 trials (six up outcomes and two down outcomes average to  $\cos 60^\circ = \frac{1}{2}$ ). It is impossible for spin angular momentum to be conserved explicitly in any given trial since the measurement outcomes are binary (quantum) with values of +1 (up) or  $-1$  (down) per no preferred reference frame and explicit conservation of spin angular momentum in different reference frames would require a fractional outcome for Alice and/or Bob.

as predicted by Maxwell's equations). If Bob is moving at velocity  $\vec{V}_b$  relative to that same light source, then he measures the speed of light from that source to be  $c$ . Here "reference frame" refers to the relative motion of the observer and source, so all observers who share the same relative velocity with respect to the source occupy the same reference frame. NPRF in this context means all measurements produce the same outcome  $c$ .

As a consequence of this constraint we have time dilation and length contraction, which are then reconciled per NPRF via the relativity of simultaneity. That is, Alice and Bob each partition spacetime per their own equivalence relations (per their own reference frames), so that equivalence classes are their own surfaces of simultaneity. If Alice's equivalence relation over the spacetime events yields the "true" partition of spacetime, then Bob must correct his lengths and times per length contraction and time dilation. Of course, the relativity of simultaneity says that Bob's equivalence relation is as valid as Alice's per NPRF.

This is completely analogous to QM, where Alice and Bob each partition the data per their own equivalence relations (per their own reference frames), so that equivalence classes are their own +1 and  $-1$  data events. If Alice's equivalence relation over the data events yields the "true" partition of the data, then Bob must correct (average) his results per average-only conservation. Of course, NPRF says that Bob's equivalence relation is as valid as Alice's, which we might call the "relativity of data partition" (Table 1).

Thus, the mysteries of SR (time dilation and length contraction) ultimately follow from the same principle as Bell state entanglement, i.e., no preferred reference frame. So, if one accepts SR's principle explanation of time dilation and length contraction, then they should have no problem accepting conservation per NPRF as a principle explanation of Bell state entanglement. Thus, NPRF

Special Relativity	Quantum Mechanics
Empirical Fact: Alice and Bob both measure $c$ , regardless of their motion relative to the source	Empirical Fact: Alice and Bob both measure $\pm 1 (\frac{\hbar}{2})$ , regardless of their SG orientation relative to the source
Alice(Bob) says of Bob(Alice): Must correct time and length measurements	Alice(Bob) says of Bob(Alice): Must average results
NPRF: Relativity of simultaneity	NPRF: Relativity of data partition

Table 1: **Comparing special relativity with quantum mechanics according to no preferred reference frame (NPRF).**

is a unifying principle for non-relativistic QM and SR, thereby addressing the desideratum of QIT in general and answering Bub’s question specifically (Figure 1).

Despite the fact that this principle explanation supplies a unifying framework for both non-relativistic QM and SR, some might demand a constructive explanation with its corresponding “knowledge of how things in the world work, that is, of the mechanisms (often hidden) that produce the phenomena we want to understand” [Salmon, 1993, p. 15]. This is “the causal/mechanical view of scientific explanation” per [Salmon, 1993, p. 15]. Thus, as with SR, not everyone will consider our principle account to be explanatory since, “By its very nature such a theory-of-principle explanation will have nothing to say about the reality behind the phenomenon” [Balashov and Janssen, 2003, p. 331]. As stated by [Brown and Pooley, 2006, p. 76]:

What has been shown is that rods and clocks must behave in quite particular ways in order for the two postulates to be true together. But this hardly amounts to an explanation of such behaviour. Rather things go the other way around. It is because rods and clocks behave as they do, in a way that is consistent with the relativity principle, that light is measured to have the same speed in each inertial frame.

In other words, the assumption is that the true or fundamental “explanation” of Bell state entanglement must be a constructive one in the sense of adverting to causal mechanisms like fundamental physical entities such as particles or fields and their dynamical equations of motion. Notice that while our account of SR is in terms of fundamental principle explanation, that does not necessarily make it a “geometric” interpretation of SR. For example, nothing we’ve said commits us to the claim that if one were to remove all the matter-energy out of the universe there would be some geometric structure remaining such as Minkowski spacetime. Furthermore, there is nothing inherently geometric about our principle explanation of Bell state entanglement in particular or of NPRF in general.

Of course we do not have a no-go argument that our principle explanation will never be subsumed by a constructive one. However, especially in light of

the unifying nature of our principle explanation, we think it is worth considering the possibility that principle explanation is fundamental in these cases and perhaps others [Silberstein et al., 2018, Stuckey et al., 2019, Stuckey et al., 2020]. We think this is especially reasonable in light of the current impasse in both QIT-based explanations of QM phenomena and in attempts at constructive interpretations. Essentially, we are in a situation with QM that Einstein found himself in with SR [Einstein, 1949, pp. 51-52]:

By and by I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts. The longer and the more despairingly I tried, the more I came to the conviction that only the discovery of a universal formal principle could lead us to assured results. The example I saw before me was thermodynamics.

Thus we are offering a competing account of quantum entanglement for any interpretation that fundamentally explains entanglement in the constructive sense. As Einstein said, this gives us the advantage of “logical perfection and security of the foundations” as our principle account could be true across a number of different constructive interpretations. And, the principle we offer, NPRF, is a unifying principle for non-relativistic QM and SR.

Per [Hicks, 2019], NPRF is a principle that is accessible (“because it is simple”) and whence we can “infer lots of truths.” Inferring “lots of truths” implies a unifying principle is superior to its subsumed constituents, since it implies (at minimum) more truths than any proper subset of its subsumed constituents. The point is, we are hypothesizing that the SO(3) symmetry with average-only conservation as an explanation of Bell state entanglement, and Lorentz symmetry with relativity of simultaneity as an explanation of length contraction and time dilation, are expressions of a deeper truth, NPRF, with seemingly disparate multiple physical consequences. It has been suggested that perhaps other unresolved phenomena in physics might be explained in a similar fashion [Silberstein et al., 2018].

The bottom line is that a compelling constraint (who would argue with conservation per NPRF?) explains Bell state entanglement without any obvious corresponding ‘dynamical/causal influence’ or hidden variables to account for the results on a trial-by-trial basis. By accepting this principle explanation as fundamental, the lack of a compelling, consensus constructive explanation is not a problem. This is just one of many mysteries in physics created by dynamical and causal biases that can be resolved by constraint-based thinking [Silberstein et al., 2018].

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